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THE POLAR PLANIMETER

AND ITS USE IN

Engineering Calculations

TOGETHER WITH

TABLES, DIAGRAMS AND FACTORS

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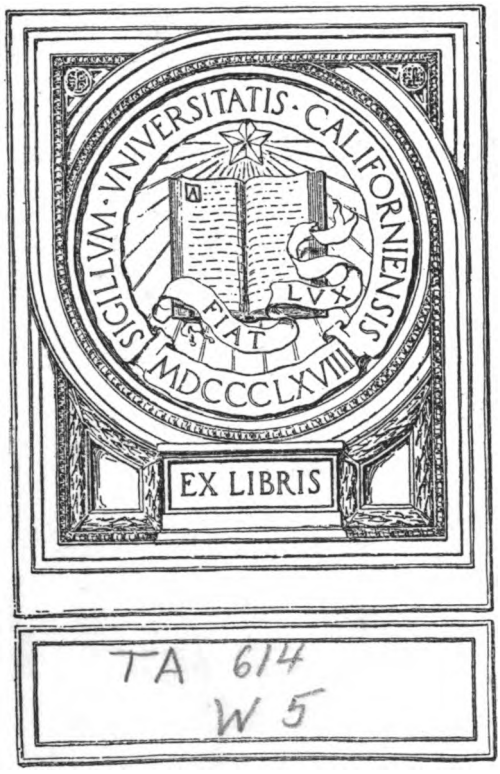

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THE POLAR PLANIMETER

AND ITS USE IN

ENGINEERING CALCULATIONS

TOGETHER WITH

TABLES, DIAGRAMS AND FACTORS

FOR THE IMMEDIATE ADJUSTMENT OF THE INSTRUMENT FOR THE SOLUTION OF PROBLEMS INVOLVING :

THE MEASUREMENT OF LARGE AND SMALL AREAS,

AVERAGE OR MEAN HEIGHT OF INDICATOR AND SIMILAR DIAGRAMS,

DETERMINATION OF CENTRE OF GRAVITY,

MEASUREMENT OF VOLUMES OF RAILWAY AND CANAL EXCAVATION,

VOLUMES IN GRADING AND DREDGING OPERATIONS,

VOLUMES IN RESERVOIR AND SIMILAR DESIGN AND CONSTRUCTION,

QUANTITIES AND VOLUMES OF BRICKWORK,

WEIGHTS OF IRON AND OTHER METALS IN CONSTRUCTION,

MEASUREMENT OF DISPLACEMENT DIAGRAMS,

ETC., ETC., ETC.

By J. Y. WHEATLEY, C. E.

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PREFACE.

The raison d'etre of the following Chapters is the direct result of the experience of the Author at the time of his first purchase of a Planimeter.

Having received the Instrument in response to an order and having read the "Directions for Use" furnished by the maker with the Planimeter, he was compelled to write the firm who supplied the instrument asking them to kindly advise him as to the best treatise on the subject that could be procured which would give him the information necessary to make an intelligent use of the instrument.

The reply made to this request was that, although their firm had acted as Agents for the sale of the Planimeter for twenty years, they knew nothing of either its capacity or method of use, and that although diligent inquiry had been made by them, they had been unable to find anything in the nature of a treatise on the subject and did not believe that anything of the kind existed—at least in this country.

This lead at once to a study by the Author, first of the theory of the Planimeter, and then of the Engineering problems to the solution of which that theory could be applied, and the following Chapters are the result of that study.

The result of the investigations thus made has been to make the author a very firm believer in the value of Mechanical Aids in Engineering Calculations, and to cause him to feel that it requires but a knowledge of the invaluable assistance these aids are capable of rendering in almost every form of Mathematical and Engineering Computation to give these instruments the prominence and importance to which their capacities so eminently entitle them.

J. Y. WHEATLEY.

COLD SPRING, N. Y.,
January, 1903.

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INTRODUCTORY.

When we consider how few of the details of professional, commercial or domestic life there are which do not to a greater or less extent require the employment of some one or more of the operations of mathematical computation for their solution, the importance of any instrument which will lessen in any degree the mental and physical labor involved in such computation is at once apparent.

In the case of the Engineer, the Scientist, Statistician and others of whose work mathematical calculations form by far the greatest part, the subject of mechanical aids in those labors assumes an importance which can not be over-estimated.

That this fact is now and has been in the past recognized is shown by the fact that the design and application of instruments for this purpose has in some one of their forms engaged the attention and thought of many of the most eminent scientists and mathematicians from the time of Briggs and Gunther to the present, and the results of the labors of these men is seen in a class of instruments which are marvels of mechanical skill and mathematical accuracy, performing the operations for which they were intended in a manner which leaves little to be desired.

While these instruments are in almost universal use abroad—France and Germany being notable examples—their introduction and use in this country has been inexplicably slow when their value and the short time necessary to acquire a working knowledge of them is considered. There is no possible explanation which can be given to account for the limited use of the instrument just referred to other than that it is due to a very limited or almost wholly lacking knowledge on the part of those most concerned both of the existence of these instruments themselves and of the invaluable aid they are capable of rendering in almost any and every form of mathematical computation. That this ignorance or lack of knowledge does exist and that it is almost unaccountably general is too evident to require proof—the limited use of the instrument being, as has already been stated, conclusive evidence of its existence.

There are several conditions which may be assigned as being the cause of this state of affairs. One cause, certainly a potent one, being the fact that in the countries named—more especially perhaps in Germany—the importance of these instruments is so fully recognized that a special course of lecture and instruction is often devoted to their theory and use, while in the crowded curriculum of our home technical institutions no attention whatever is given to them, a condition which is usually the case with any subject which has any chance whatever of being acquired in any other way. In a few—and that but very few—institutions in this country some instruction may be given in the subject, but in no case is it more than superficial, and always entirely inadequate when the great importance of the subject is considered.

Another assignable reason lies in the fact that there seems to exist with a great many people a vague and wholly unreasoning distrust of either the accuracy or the practical value of any instrument designed to perform a mental operation. The idea seeming to be prevalent that any

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result obtained in any such manner should be regarded with suspicion: that even granting such results to be of sufficient accuracy to be of use, that the instrument capable of performing it must of necessity be extremely complicated, requiring a large amount of time and study for its intelligent use, and that even when a working understanding of it had been acquired the saving of time and labor effected by its use would be much too small to warrant the time spent in its acquirement.

To the above two causes assigned as responsible for the limited use of mechanical aids to mathematical calculation may be added a third which is the almost total lack of literature dealing with the subject under discussion: a literature, if so it may be called, utterly lacking in both extent and quality when compared with the importance of the subject with which it is concerned. The instruments being for the most part of foreign design and make, it follows that most of the literature concerning them is also in a foreign language, and while the results of studies and investigations of many foreign mathematicians have appeared in book or pamphlet form their value to the student in another country must necessarily be limited by the linguistic attainment of the student who desires to avail himself of those results.

There of course remains to us the "*Directions for Use*" usually furnished by the maker, but unfortunately in the great majority of cases not only do these "directions" utterly fail to direct but they fail entirely to do justice to the instrument described and often do not even hint at many of its most valuable applications—thus doing an injustice both to instrument and owner.

The remarks thus far made while applicable to almost every form of instrument of the class under discussion are particularly true as applied to the particular instrument we are about to discuss, and much that can be said of the Planimeter in this connection will apply equally to other instruments of which it is a type.

Contrary to the usual belief and in common with other instruments of its class, a knowledge of the theory or principle of the Planimeter is not an essential either to an ability to use it or to obtain accuracy in results when applied to the solution of those problems to which it is particularly adapted. An ability to use the Planimeter can easily be acquired in a very few moments, but at the same time the time and effort necessary to acquire the additional knowledge of the mathematical and mechanical operations involved in its theory and construction will be well repaid, not only in the increased appreciation of the instrument itself and greater confidence in the accuracy of results obtained by its use, but also in the ability to apply it to problems involving unusual conditions and to pass intelligent judgment on instruments of like nature and design.

For this reason the demonstration of the theory of the Planimeter is given in as clear and concise a manner as possible and from it are deduced the formulæ by which the data for the adjustment of the instrument to meet the varying conditions involved in its many applications is obtained. In all demonstrations the use of the higher mathematics has been purposely avoided and all mathematical operations are reduced to their simplest forms.

The examples given as illustrating the application of the Planimeter to the solution of the various problems of engineering have been selected with a view to having each such example a type of a class of problems requiring similar operations for their treatment, and the explanations are made as general as possible to enable the intelligent use of the instrument in every other problem belonging to that particular class.

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In the treatment of each example given, the adjustment and method of use of the Planimeter as applied to the special problem involved is concisely and logically given: the particular operation involved being deduced directly from the general theory of the Planimeter. The adjustment and use of the instrument in each example is clearly described and is followed by a discussion of the relative and actual degree of accuracy attainable in each case, together with notes on the method of operating and the securing of a maximum efficiency.

The Tables have been made as complete as possible and the effort has been made to include in them the data for almost all of the more commonly occuring problems of engineering practice. A full description of the Tables with their arrangement and method of use is given under the head of EXPLANATION OF TABLES and need not be repeated here.

Both in the treatment of the theory and use of the Planimeter and in the Tables the effort has been made to make all that follows not alone a Treatise on the Planimeter but also an Office Book for constant use, enabling the data for the adjustment of the Instrument for any operation to be at once obtained without calculation, and the entire arrangement has been with that end in view.

In conclusion it can be said not only of the Planimeter but equally of almost every instrument of the class to which it belongs, that a knowledge of its invaluable capabilities and of the enormous saving in time and labor effected by its use is the only requisite to make it the co-laborer of the Engineer in almost every detail of his professional life.

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EXPLANATION OF TABLES.

As has already been stated, the Tables are intended to give at once by inspection and without further calculation the data necessary to adjust the Planimeter for use in the solution of most of the more frequently occurring problems arising in the Engineer's practice. The Tables have been calculated and checked by both Logarithmic and Slide Rule methods, and contain all the factors necessary for adjustment of the Planimeter for any operation. Columns of factors by means of which the accuracy of the results obtained by the instrument can be readily checked have also been added to facilitate the work of the calculator, and to make the Tables as complete and useful as is possible. As the derivation and use of the factors contained in the Tables are fully explained in the following chapters it will be unnecessary to repeat it here, but a brief explanation of the general arrangement of the Tables will perhaps aid in facilitating their use and give a clearer idea of their arrangement.

As in many cases the scale of a drawing is given in the form of a ratio instead of being expressed as a definite number of parts to an inch, Column 2 has been added, so that the equivalent ratio of any scale expressed in inches and fractions of an inch is at once seen.

Columns 4, 7 and 8 in many of the Tables might perhaps have been omitted as not being strictly essential to the use of the Tables, but they serve a very useful purpose not only as a guide and aid to any desired extension of the Tables, but are also valuable both as a check on the accuracy of such additional calculations and on the position of the decimal point—a matter of the greatest importance where the numbers dealt with contain many figures.

It will be seen that in all of the Tables blank columns have been added (as Cols. Nos. 15 and 16 of Table No. 1). The reason for this is that although two Planimeters may have been made by the same maker and intended by him to be exactly alike in every detail, the delicacy of the instrument is such that there always exists between them a slight variation, due to process of manufacture and to the impossibility of two pieces of delicate mechanism being exactly alike in every respect. This variation, which is a constant for any particular instrument, is shown by a slight difference in the numerical value of the Setting and Constant between the values for these factors given in the Table for any given scale and the values given for these same factors on the card usually furnished by the maker with each instrument. To illustrate : take the values given for the Setting and the Constant in Table No. 1 for the scale 1:1000 which are 302.8 and 17420 respectively. Let us suppose that the values of the Setting and Constant for that same scale given on the card enclosed in the case of any particular Planimeter to be 303.2 and 17430 respectively. The differences between these two sets of factors are $303.2 - 302.8 = 0.4$ in the Setting and $17430 - 17420 = 10.$ for the Constants. These two values then, 0.4 and 10.0, represent the physical difference between the two instruments mentioned above, and if that instrument which has 302.8 for the value of the Setting and 17420 as the value of the Constant has been used in the preparation of the Tables, all that is necessary to be done to make the Tables applicable to the Planimeter having 303.2 for its Setting and 17430 for its Constant for the same scale will be to add .4 to the

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number given in the Tables for the Setting for *each* scale, and 10 to the number given for each Constant, placing these new values in the Columns left blank for this purpose. By this simple operation, which will take but a few moments to perform, we can make the entire set of Tables applicable to any particular Planimeter and the Tables will be in every respect the same as though that particular instrument had itself been solely used in their preparation.

In some forms of Planimeters the zero graduation of the Tracer Arm, instead of being at the Tracer end of the arm is at or near its other extremity. In this case it is evident that the values of the Settings are reversed, the higher values indicating a *shorter* Tracer Arm and the lower values a *longer* one. Formulae for finding the equivalent value of a Setting or Constant given for one form of graduation in terms of the other form are given later, but to facilitate the use of the Tables and increase their adaptability to every form of Planimeter, Columns Nos. 11 and 12 have been added, and the values given in these added columns are the equivalent values of the Settings and Constants given in Columns Nos. 9 and 10. Columns 9 and 10 being for those Planimeters having the zero of their graduation at the Tracer end and Columns 11 and 12 the equivalent value for those instruments in which the Zero of the graduation is at the opposite end of the Tracer arm.

As has already been stated, the Tables have been carefully computed and checked by both Logarithmic and Slide Rule methods, and are believed to be without error. Owing, however, to the large number of figures involved in the calculation of each factor, there may be a slight possibility of error, more especially in the position of the decimal point, but they are few if any, and easily discoverable by use.

Since there can be any number of Settings for any given Scale with their corresponding Vernier Units, that Setting has been selected which can be used for a number of different scales, making the Vernier Unit the variable, as by this method the same Setting can be used for various scales, thus lessening the necessity for frequent adjustments. While perhaps in a few cases a different Setting than that given in the Tables might allow of a simpler Vernier Unit, that advantage is more than offset in the method adopted by considering the Setting constant for a certain set of scales, and allowing the Vernier Unit to vary.

The value of the Actual Vernier Unit for any given scale is given in Column 5, and will be found very convenient when the *actual* area of any figure drawn to that scale is desired: the actual area being in each case equal to the product of the Reading for that figure and the Actual Vernier Unit. In this way the actual area is at once obtained without the necessity of resetting the Planimeter to the setting for the Scale 1″ = 1″ and retracing the figure. The ease with which the actual area is thus obtained admits of a very easy check on the accuracy of the results obtained by the use of the Planimeter, since the check consists simply in multiplying the actual area thus found by the Unit of Area of the Scale to which the figure is drawn.

Column 4 of Table VII and VIII gives the number of Cubic Yards represented by one actual Square Inch of cross-section area of any given prismoid for the given Scale and length, while Columns 7 and 8 of the same Table give the number of Cubic Yards in a prismoid whose length is the same, and the sum of whose end areas plus four times their mean area is an actual area of 15.5 Square Inches, or the area of the Test Plate. This will be more clearly understood after reading the description of the use of the Planimeter in Earthwork calculations which is given later.

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As all of the Tables with method of using them together with detailed descriptions of the derivation and use of the factors entering into them are fully explained in their proper place further description need not be given here.

The diagram given on Plate II will be found very useful in many ways—and if accurately drawn and on a fairly large scale, with reference to the particular Planimeter with which it is to be used, will give the Reading for any given Setting or the Setting necessary to produce any desired Reading when tracing an actual area of 10,000 Square Millimetres with a degree of accuracy quite sufficient for any but the finest work. It can be prepared either by calculation, or the data for plotting the curve may be acquired by trial: the latter method being however preferable, as no instrument correction need then be applied to the factors obtained from it. In the diagram the curve BB is for the instrument we are using for illustration, while AA is the curve of Settings for an instrument having the Zero of the Tracer Arm graduation at the Tracer end of the Arm. Its method of construction and manner of use are evident, and will be referred to in a later Chapter.

As has already been stated the intent in the following demonstrations, descriptions and tables has been

1. To clearly explain the principle involved in the design, construction and operation of the Planimeter.
2. To show how the Planimeter by means of these principles can be made an aid of almost any incalculable value to the Engineer in almost every detail of his professional practice, not only by lessening the mental and physical labor inseparably connected with such practice, but also furnishing by its use results of a degree of accuracy in many instances impossible by any other means.
3. To facilitate the use of the Planimeter by tabulating the factors necessary for its adjustment for most of the more frequently occuring operations of Engineering practice, thus saving time and labor otherwise expended in this calculation.

While the Tables will be found to be very complete, and to cover a wide range of operations, they are easily capable of expansion along any particular line, if so desired, and much of the data necessary for such desired expansion can be taken from the factors given in the Table devoted to that particular class of problem.

Refore beginning the actual measurement with the Planimeter in any problem, it is well to check the accuracy both of the Instrument and its Setting, and this is very easily accomplished for every operation by the arrangement of the factors given in the Table, the operation being as follows :

Having adjusted the Planimeter to the Setting given in the proper Table for the particular operation to be performed, the instrument is brought to a Zero reading and is then caused to trace the 15.5 Square Inch area of the Test Plate. At the end of this tracing, which should take but a moment to do, the Reading of the instrument is taken, If the Planimeter is in good adjustment and the tabulated value of the Setting is without error, this Reading should be exactly the same as that given in Column 8, under the heading "Reading" in the Table used. As the exact number of Vernier Units which the Planimeter should record when in accurate adjustment is given in this Column for every scale in all the Tables, the values of this method of checking is apparent.

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CHAPTER I.

Planimeters in General.

Of the many instruments which have been devised at various times to facilitate the long and tedious calculations which claim so large a share of the time and labor of the Engineer, the Polar Planimeter is easily the most valuable.

Forming one of a class of similar instruments usually designated "Planimeters," and standing midway between the simple Planimeter with its limited range and restricted field of operation and the complicated and costly Integraph and like instruments, the Polar Planimeter can advisedly be said to be adapted to the solution of almost every problem arising in the Engineer's practice.

Planimeters Defined.

As ordinarily defined the Planimeter is an instrument by means of which the area of any plane figure coming within its range can be accurately measured without calculation and regardless of the shape or irregularity of its outline.

Were the capacity of the instrument limited to the single operation of the accurate measurement of plane areas, as just stated, the Planimeter would still be one of the most valuable, if not THE most valuable of the Engineer's mechanical assistants, but when we consider that not only will the Polar Planimeter perform this operation with an accuracy and rapidity unequalled by any other known means, but that it has in addition a range of application so wide as to include almost EVERY form of operation incident to Engineering work, we are able to gain some adequate appreciation of the value and importance of the instrument.

So much has already been said on this subject in the introductory chapter that it need not be further spoken of here, the following chapters describing these practical applications furnishing abundant proof that the statements there made by no means exaggerate either the capacity or ability of the instrument.

History.

To whom the credit belongs for the invention of the Planimeter seems to be a disputed question.

While claims to that distinction have been made by and for several well-known and distinguished mathematicians who have devoted their time and attention to the subject, the best evidence goes to prove that the Planimeter, instead of being the product of the inventive skill of any one mind, is rather the combined results of the investigations of several, each working independently and often entirely unknown to the others—a fact which has characterized the production and perfection of many of the most important of modern inventions.

The employment of a wheel of known circumference rolling along a curved or broken line to determine its length is very old—mention

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being made as having been thus used by the earliest of Egyptian mathematicians—and the principle is still in use in the present so-called Opisometer and other instruments of similar nature.

Its application as a factor in an instrument for the measurement of the areas of plane figures seems, according to Prof. Shaw, to have been first adopted by Hermann of Munich about the year 1814.

The first completed Planimeter of which we have definite record appears to have been invented by Oppikofer and to have been exhibited by him in Paris in 1836.

A patent on a Planimeter was granted in 1849 to Wettli and Starke, who invented the instrument which still bears their names.

Other investigators who have devoted time and labor to the design and perfection of the instrument are Profs. Miller and Lorber of Loeben, Lämmle of Munich, and Bouniakovsky of St. Petersburg.

Since then the subject has received the attention of a large number of scientists and mathematicians, among the most prominent of whom are Mr. Coradi of Zurich and Prof. Amsler Laffon of Schaffhausen, who have both added improvements and extended the field of usefulness of the instrument until the Polar Planimeter of to-day is a marvel of mechanical skill and mathematical accuracy, performing the operations for which it was designed in a manner which leaves little to be desired.

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CHAPTER II.

The Polar Planimeter.

As has already been stated, all forms of the simple Polar Planimeter of whatever make are essentially the same in principle and in the general relation and arrangement of parts. The various forms of the instrument differ from each other only in some one or more of the minor details of mechanical construction and in degree of refinement of finish.

In the best and most accurate Planimeters every device known to mechanical skill has been utilized to give to these instruments the delicacy of adjustment and operation so necessary to their accuracy and maximum efficiency while at the same time detracting nothing from their requisite strength and ability to withstand the wear and tear of intelligent continued service.

Essential Parts of the Instrument.

The Polar Planimeter, Fig. 1, Plate I, consists essentially of the arm *F P*, known as the *Polar Arm*, the *Wheel W* termed the *Integrating Wheel*, and the graduated arm *R S* known as the *Tracer Arm*, all of polished german silver. The lengths of these two arms as well as the diameter of the wheel are entirely arbitrary and differ slightly in Instruments of different makes. Nor is it essential that the dimensions of these parts should bear any definite relation to each other than that which experience has proved to be the most suitable for any given form of instrument. The usual length of both Polar and Tracer Arms is from six to eight inches with a diameter of Integrating Wheel of perhaps three-quarters of an inch, these dimensions having been found to be the most convenient.

Polar and Tracer Arms.

Both Polar and Tracer Arms are usually square in section and either made of rectangular tubing or have the under part of the Arm cut away in order to make them as light as possible consistent with the requisite strength and rigidity. One end of the Polar Arm *P* terminates in what is called the *Pole* of the instrument, and *Polar Planimeters* are usually divided into two classes, termed respectively *Needle Pole Planimeters* and *Ball Pole Planimeters* according to the form and arrangement of this end of the Polar Arm. Of the two forms of Pole the Ball Pole is the one adopted for the finer grades of Planimeters; the Needle Pole being used generally in those instruments which do not have the graduated Tracer Arm and on which only simple ratios are indicated as will be explained later.

Pole and Its Forms.

In the Ball Pole Planimeter, which is the one we are discussing, the end of the Polar Arm *P* carries a short rod of polished Steel permanently attached to the Arm. This short rod terminates in a ball which fits accurately into a conical receptacle in the top of the *Pole Weight* and without shake or play, the ball and its receptacle forming a fixed center around which the entire instrument can freely turn.

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The *Pole Weight* carrying this conical recepticle may be either round or square in shape and is usually made of some heavy metal such as lead or brass to prevent any accidental movement of the *Pole* while operating the Planimeter.

In some forms of the instrument the Pole is held more securely in its receptacle by means of a small cylindrical weight which is slipped over the prolongation upwards of the short steel rod already mentioned. In the form under discussion the Polar Arm is extended beyond the Pole and carries the Weight B at the further extremity of this extension as shown on the drawing. This arrangement not only serves the desired end of holding the Pole securely in its seat but has the further advantage of acting as a balance to the weight of the entire instrument and relieving the Wheel of all unnecessary pressure on the paper.

The other end of the Polar Arm is connected with the frame or Carriage C of the Planimeter by a fine pivot-joint at F which allows of perfect freedom of movement of Tracer Arm and Carriage about F as a center.

From this description it will be seen that the Tracer T revolves about the Carriage Pivot F as a center, while the Pivot F, and in fact, the instrument itself revolves about the Pole P as a fixed center with the length F P of the Polar Arm as constant radius.

Tracer Arm and Its Graduation.

In all the better forms of Planimeters the Tracer Arm R S is graduated for almost its entire length into some scale of equal parts and with all its principle divisions and sub-divisions plainly marked. The principal use of this graduation is to allow of the recording of any position or point on the Tracer Arm and the certainty of any subsequent correct locating of that point when desired; it also furnishes a means of obtaining the exact distance measured along the Arm between the Tracer and the point in question.

The Scale or form of Graduation of the Tracer Arm varies with the different forms and makes of Instrument. In some Planimeters the graduation is in inches and decimals of an inch ; in others the division is into half millimeters and fractions while in another form the Arm has been divided into fiftieths of an inch as being a sort of compromise between the Metric and English systems.

For reasons which will appear later a graduation into one-half millimeters as the principal divisions, with each such division subdivided into ten equal parts, making twentieths of a millimeter as the space between divisions lines, is perhaps preferable to any other form.

The beginning or Zero of the graduation may be at either end of the Tracer Arm, but placing the Zero at the Tracer and having the numbers reading from left to right has many advantages not possessed by the other system of notation.

Carriage, Carriage Vernier and Attachments.

The frame or *Carriage C* of the instrument carries the *Integrating Wheel W* together with the recording mechanism and the necessary adjusting screws. As already stated the Polar Arm is connected to the Carriage by the pivot F, while the Tracer Arm passes through sleeves or bearings on the Carriage which allows the Arm to slide backward and forward or to be rigidly clamped to the Carriage at any desired point by means of the binding screw L. Attached to the Carriage is a Vernier V called the *Carriage Vernier*, so situated as to allow the graduation of the Tracer Arm to work along its edge, and by its use the smallest division of

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the graduation is still further subdivided into ten smaller parts which in this case are one two-hundredths of a millimeter. By this arrangement the Zero of the *Carriage Vernier* can be brought very near to any desired point on the graduated Tracer Arm, the binding screw L tightened and the exact adjustment made by means of the slow motion screw M.

The exact point on the Tracer Arm to which for any given operation the Zero of the Carrige Vernier is required to be brought or *set* is, as has already been mentioned, both definitely located and recorded by expressing its position in terms of the unit of the scale of equal parts engraved along the edge of the Arm, and the point thus located and recorded for any given operations is called the *Setting* for that operation.

Settings.

In the case of Planimeters the Zero of whose Tracer Arm graduation is at the Tracer, the *Setting* in any case is evidently equal to and expresses the distance between the Tracer and the Zero of the Carriage Vernier. As the *Length of Tracer Arm, which is always the distance between the the Tracing Needle T and the Carriage Pivot F*, is an important factor in all Planimeter calculations, this form of graduation has the important advantage of allowing this length to be at once obtained from the Setting by the simple operation of subtracting from the Setting the distance between the Carriage Vernier Zero and the Carriage Pivot F, which is a constant for any given instrument.

By placing the Zero of the Tracer Arm graduation at a distance from the Tracer T equal to the distance between the Zero of the Carriage Vernier and the Carriage Pivot *F* the Setting is evidently in every case also the expression of the length of the Tracer Arm or the distance F T. Coradi has adopted this practice with his Compensating and other high grade instruments with most advantageous results.

Tracer Rests and Adjustments.

At the other end of the Tracer Arm from the Carriage is fixed the *Tracing Needle* or *Tracer* as shown in Plan in Fig. 1, and in elevation in Fig. 3 of Plate I.

This Needle or *Tracer* is of finely polished steel, brought to a very fine point at its lower end, and so arranged as to permit of very accurate and delicate adjustment sideways by means of the two screws *H H* shown on the drawing.

Attached to the Needle is the *Tracer Rest E.* which is adjustable in height and which forms the point of support instead of the needle itself. By proper adjustment as to height it allows of the point of the Tracer being brought very near to the surface of the drawing paper, hovering over it but not touching it. The Rest also acts as a handle and guide by means of which the Tracer can be accurately guided along any desired path or along the outline of any figure, and greatly facilitates the accurate tracing of any bounding line. In the most improved forms of Planimeters the form of Rest is modified from that just described, and is shown in its improved form in Fig. 2 of Plate IX. One important improvement consists in enclosing the Tracing Needle in a spiral spring in such a manner that while during the ordinary operations of tracing the needle does not touch the paper, by pressing the top of the needle with the finger the needle is forced down so that its point enters the paper and makes a fine but very distinct hole which remains as a permanent mark at the starting point or at any other point it may be desirable to preserve. On removing the pressure of the finger the needle at once resumes its adjusted position.

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Integrating Wheel and Recording Mechanism.

The *Integrating Wheel W* with its attachments and adjusting mechanism form by far the most important part of the Planimeter and on their condition and on the delicacy and accuracy of their adjustment depends to a very large extent the value of the Instrument.

The Wheel itself is usually about three-quarters of an inch in diameter and is made of very highly polished metal, hard nickel being the material usually selected owing to its freedom from tendency to rust. The Wheel is mounted on a spindle or axis of hard steel having at each end very fine conical points which fit into conical receptacles in bearing bolts—which are set into the Carriage frame at both ends of the spindle. As the accuracy of the Planimeter demands that this axis of the Integrating Wheel shall be exactly parallel both to the surface on which the wheel moves and to the Tracer Arm of the instrument, these bolts which form the bearings of the axis are capable of very accurate adjustment both horizontally and vertically by means of very fine adjusting screws in the Carriage or frame which act on the bolts and bring them to the desired position.

It is also a strict essential that the axis shall be so adjusted in its hangings that friction shall be reduced to a minimum and that the wheel shall revolve with the greatest possible freedom but without perceptible play or shake in its bearings.

To the wheel and its axis is attached the recording mechanism of the instrument by means of which the total number of revolutions and fractions of a revolution made by the Integrating wheel during any given operation may be accurately measured and recorded.

Attached to the Wheel and of slightly less diameter than the rim of the Wheel is a drum, usually of white celluloid. The circumference of this drum is accurately divided into ten equal parts and the lines marking such parts are properly numbered consecutively from 0 to 9. These ten equal parts are subdivided each into ten other equal parts marked by smaller marks, thus dividing the circumference of the Wheel into one hundred equal parts. Each one of these one hundred divisions is marked by a black line cut on the white drum.

Attached to the Carriage in such a manner that the edge lies along the graduated edge of the drum is a white celluloid plate having a vernier cut along its edge. By means of this vernier the space between two consecutive lines on the drum is divided into ten other equal parts. This arrangement of the graduated drum and its vernier divides the circumference of the drum into one thousand equal parts and since one complete revolution of the Integrating Wheel causes the drum to move over one thousand of the equal parts into which its edge is divided, it follows that the drum and its vernier will give at once the fractional part of any revolution made by the wheel in terms of these parts.

At *U* on the axis of the wheel is cut a worm screw which engages with a pinion which forms the axis of the small graduated white celluloid wheel *O*. This worm is so arranged that one complete revolution of the Integrating Wheel and its axis will cause the wheel *O* to make one-tenth of a revolution about its center, and since the circumference of the wheel O is divided into ten equal parts it is evident that the wheel O will record the number of whole revolution of the wheel *W* made during any operation.

It is then evident that the wheel O together with the graduated drum and its vernier will give at once and accurately the whole number of complete revolution and the tenths, hundredths and thousandths of a revolution made by the Integrating Wheel W during any operation of the

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Planimeter. It further follows that if we know the circumference of the Wheel W, the number of revolution and fractions of a revolution thus recorded will give us accurately the length of any path rolled over by the wheel during the operation which caused those revolutions.

Vernier Units, Relative and Actual Vernier Units.

The one one-thousandth of the Revolution of the Integrating wheel *W* for any operation is called the *Wheel Unit* or *Wheel Vernier Unit* and the number of such units recorded by the recording wheels during any given operation is called the *Reading* for that operation and will always be designated by the letter r.

In all operations of the Planimeter in the solution of any problem a definite numerical value is assigned to the Vernier Unit. The value thus assigned is called the value of the *Relative Vernier Unit* or the *Relative Vernier Unit* and is dependent in any given operation on the conditions of the particular problem of which that operation is a part. These assigned values for various conditions are given in the Tables and their significance and use will be clearly understood later in the discussion.

Adjustment and Care of the Instrument.

The adjustments of the Polar Planimeter are few and simple but the accuracy of both instrument and its operation depends upon the perfection of those adjustments and upon the delicacy and care with which they are effected.

That condition on which more than any other the accuracy of the Polar Planimeter depends is that the axis of the Integrating Wheel W shall be exactly parallel to the Tracer Arm R S. The importance of this parallelism is readily shown not only by actual trial but also by reference to the formulae deduced in the mathematical demonstration of the theory of the Planimeter given in a succeeding chapter and from which the effect in kind and extent of any deviation from parallelism is at once determined.

Co-important with this condition is the further one that the axis shall turn in its bearings with the most perfect freedom and delicacy possible and with a minimum amount of fricture but without play or shake of any kind.

The adjustments of the axis to cause it to conform to these two given conditions are easily effected by means of correction and binding screws in the Carriage C at both ends of the axis which give movement in any desired direction to the hardened bolts which form the bearings of the conical ends of the axis.

An attempt to mimimize the errors in the operation of the Polar Planimeter due to the two causes just mentioned has resulted in the form of instrument called the Compensating Planimeter by its inventor, Mr. Coradi, by which the errors while present in the result of a single operation, are practically eliminated or *compensated* by taking the *mean* of the results of *two* operations in which the effects due to nonparallelism are in one operation taken positively and in the second operation negatively. This instrument being the highest form of the class of Planimeter under discussion will on account of its importance in this connection be taken up and described in detail later.

After a long period of use or as the result of accident the axis of the wheel may appear sluggish and to have lost the delicate sensitiveness and freedom of movement so essential to the highest efficiency of the Planimeter. Should this be due to a dulling or injury to the conical ends

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of the axis the only remedy is to place the instrument in the hands of a competent instrument maker who can easily restore it to its original condition. In most cases, however, this dullness or sluggishness is due to the oil which has been applied to the bearings of the axis becoming congealed or hardened, in which case the defect is easily remedied by loosening the set screws which control the bearing bolts and introducing with a fine aluminum point a drop of benzine into each bearing and then revolving the axis quickly in both directions. This will dissolve or soften the congealed oil which can then be removed by a very soft cloth. After thoroughly cleaning the bearings a drop of the finest watch oil should be introduced to each bearing by the metal point and the axis then readjusted.

In using the Planimeter great care should be taken to avoid any injury to or bending of either Tracer Arm or Needle, as the effect of such bending would be to seriously impair the accuracy of the instrument and its results. The rim of the Integrating Wheel should be carefully guarded against injury of any kind, as the slightest dirt or even speck of rust on the rim would effectually interfere with the accuracy of its operation. For this reason the Wheel should never be turned or even touched by the finger, and after use the instrument should be carefully wiped with a very soft chamois skin and the rim of the Wheel allowed to run several times over the chamois laid flat on the table or drawing board.

While every device to increase the sensitiveness, and hence the efficiency of the Planimeter, has been utilized, at the same time due care has been taken that the strength and rigidity of the instrument has not been impaired, and that with proper care it shall be capable of long continued and effective service and with small liability to injury if used with the precautions mentioned.

Test Plate and Its Uses.

Enclosed in the case with each Plaimeter will be found what is known as a Test Plate. This Test Plate may have the form of a circular metal disk having a groove cut along its edge as shown in Fig. 2 of Plate I, or it may consist of a metal strip of the form shown in Fig. 3 of Plate IX. The intent or purpose of the Test Plate is to furnish an accurately known constant area by means of which the accuracy of the Planimeter and its adjustments can be tested. In the case of the circular grooved disk the area of the circle of which the groove is the circumference is accurately known, and by placing the point of the Tracing Needle in the groove, this known area is accurately traced by moving the Tracer along the groove until it has made a complete circuit and returned to the starting point. The other form of Test Plate consists, as has been said, of a short metal strip or plate. At one end of the plate, and held in position by means of the large screw head shown, is a very fine needle point which projects a short distance from the face of the plate. Along the center of the plate, and at different distances from the needle point, are very minute conical receptacles into which the point of the Tracer fits without play. These points are so spaced that when the needle point at the end of the strip is firmly pushed into the paper the distance from the needle point to any given receptacle is the radius of a circle, the area of which is a known quantity; the circle itself having for its circumference the path of that particular receptacle when the strip is revolved about the projecting needle point as a center.

The method of using the Plate is very simple and is as follows : Having pressed the needle point of the Test Plate firmly into the paper, the Tracer of the Planimeter is placed in the receptacle selected, and either a

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mark made on the paper opposite a line cut in the outer end of the Plate or a pin is pushed into the paper to mark the point of beginning. The Test Plate is then caused to revolve about the needle point at its end by pushing the other end of the plate with the finger until the mark at the end of the plate having made a complete revolution about the needle point has returned to the marked starting point. It is evident that by this operation the receptacle and the Tracer have traced the circumference of a circle the area of which is accurately known and the Test Plate has fulfilled the purpose desired.

The number usually found engraved at each receptacle on the Plate is in most forms of plate the actual area of the circle whose circumference is the path travelled by that point during the revolution of the plate; this area is usually expressed in terms of the Unit of graduation of the Tracer Arm which in most cases is square millimetres.

When using this form of Test Plate in the manner described the plate is often moved through its revolution by taking hold of the Tracer with the fingers. This practice is objectionable for several reasons as not being conducive to the greatest possible accuracy during the operation, and a much better plan is to keep the Tracer firmly in the receptacle during the movement by placing small weights on the Tracer Arm and then to cause the revolution by moving the Test Plate with the fingers—not touching the Tracer at all with the fingers during the entire operation.

Of the two forms of Test Plate just described the second or strip form is preferable to the grooved disk for a number of reasons and is the form usually supplied with the best grade of Planimeters.

It often happens that the area to be measured may be on an old map drawn on wrinkled or torn paper and otherwise not conducive to accurate working of the Planimeter. In this or any similar case the path taken by the Wheel while the Tracer traces the outline of the given area can be easily determined by trial and a sheet of paper having a proper surface can be laid down in such a manner that the entire travel of the Integrating Wheel shall be on this paper and not on the objectionable surface.

Influence of Surface on Wheel Movement.

As the revolution of the Integrating Wheel and hence the entire working of the Polar Planimeter is due to the frictional contact between the rim of the Wheel and the surface of the paper or other material upon which the Wheel moves, it is evident that the nature of this surface must be a matter of the greatest importance in every case where a high degree of accuracy is desired. One great advantage possessed by the Rolling Spherical Planimeter and the Precision Disk Planimeter and other of the higher form lies in their complete independence of the nature of the surface upon which they are operated. In the case of the Rolling Spherical Planimeter the movement is produced by very heavy toothed wheels which roll over the surface and on whose motion inequalities and irregularities which would be fatal to the accuracy of the Polar Planimeter have little or no appreciable effect, while in the Precision Disk form the Integrating Wheel rolls over and receives its motion from a polished metal disk of uniform surface and free from any injurious influence.

In the case of the Polar Planimeter uniformity of surface is a very strict essential to accuracy since the movement of the wheel which is partly slipping, partly rolling over the surface should have identically the same conditions of surface at every point in its path in order that the influence due to the surface may be of the same kind and extend throughout

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its entire operation. In fact uniformity of surface is of more importance than the nature of the surface itself, since very accurate results are obtainable with any of the papers or materials ordinarily used for drawing and map making purposes. Those drawing papers having a medium smooth surface finish give perhaps the best results and the paper need not be glued or stretched on the board provided it lies smoothly and without wrinkles.

Care should be taken that the drawing board or table on which the paper lies should be perfectly level, true and free from any irregularities.

Most Favorable Position.

The best position of the Polar Planimeter with relation to the outline of the figure to be traced is shown in Fig. 3, of Plate V. In general the "favorable" position of the Instrument with relation to any figure may be determined by the following general rule: *Place the Tracer at the center of the figure to be measured and place the pole P in such a position that a line drawn from the Pole P to the rim of the Wheel W will be at right angles to the Tracer Arm.* This position is indicated by Fig. 3 of Plate V, as mentioned above.

Point of Beginning.

The starting point of the Tracing should be at or near the point as shown on the diagram. This point is the intersection of the "Constant Circle" or "Base" with the outline of the figure to be traced and is selected as the most favorable position for both beginning and ending the operation of tracing owing to the fact that it is at this point of the outline there is the least amount of movement of the wheel for any given movement of the Tracer, and hence at this point that any accidental error or deviation of the Tracer will have its minimum effect on the movement of the Wheel.

A further consideration in selecting the best position of the Planimeter with respect to any area to be traced is that the Instrument should be so placed that in tracing its outline no important side of the outline shall be parallel to or near the "Base." The reason for this is evident from the conditions named above as governing the choice of a starting point for the tracing.

As has already been stated the description of the Polar Planimeter just given will apply in all essential particulars to all forms of the Polar type of Planimeter, as the theory and operation are identical in all.

In the higher types of the instrument such as the Rolling Spherical, the Precision Disk Planimeter, the Spherical Polar, the Integraphs and others of like character—some of the more important of which are described later in our discussion—other principles are involved in construction and theory which admit of the performing of a class of mathematical operation impossible to the simple Polar form. But in operation possible to both Precision and Polar types the more complicated nature of the construction and theory of the so-called Precision instruments and the additional precautions taken in their design and operation to avoid every possible source of error must necessarily permit of the obtaining of results by their use of a relatively higher degree of accuracy than is obtainable by the simpler form. While this may be of advantage in the case of a small class of problems necessitating an unusual degree of accuracy in the instrument, in the great majority of cases if the adjustments as described above are carefully made and the conditions for accuracy as given

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are closely observed, the Polar Planimeter will give results of such a degree of accuracy as to be far within the error allowable in any given problem and far higher than would be possible by any other known means.

In some forms the Polar Planimeter either by change in some minor details of mechanical construction or by a special attachment the instrument is made especially adaptable to some one particular operation which by this means it is enabled to perform with greater facility without lessening its value for other operations. These special forms will be described in their proper place when describing the use of the Planimeter in its practical application involving the operation in question, as will also further notes on the adjustments, methods of operating and conditions requisite for maximum efficiency in the use of the instrument.

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CHAPTER III.

Theory of the Polar Planimeter.

The mechanical construction of the Polar Planimeter and its various essential parts having been described in detail in the preceding chapter, the general theory of the instrument will now be taken up and the mathematical relation of the different parts to each other and to the operation of the Planimeter will be described in as simple and clear a manner as possible.

This demonstration together with the description of the instrument just referred to should show not only how the instrument described mechanically performs the operations demanded by the theoretical considerations involved in its principle and operation, but also the relation each essential part of the instrument bears to each other part and to the Planimeter as a whole, the relative importance of each such part, and the nature and extent which mechanical defect or non-adjustment of any part of the instrument would have on the theoretical degree of attainable accuracy of operation and result.

Before taking up the strictly theoretical consideration of the Polar Planimeter however it may prove of service in making the following explanations and demonstrations clearer to devote a few lines to the subject of the theory of Planimeters in general.

Theory of Planimeters in General.

While a full discussion of the general theory of Planimeters and of the special theoretical considerations involved in the various particular forms of the instrument would be of much interest and especially so to the mathematician and mechanical expert, lack of space will necessitate the limiting of such detail discussion to the particular type we have selected for description and illustration in the following chapters, confining reference to other forms to description only.

The general theoretical principles involved in the design and operation of the Planimeter class of instruments while differing widely in some forms of the instrument—notably in the higher types—are essentialy the same in most of the simpler forms and in the particular Polar form to which the Planimeter we are discussing belongs.

For this reason, while particular instruments may vary in some one or more points of mechanical detail or arrangement of parts, the following demonstration and statement of the theory of the Polar Planimeter will be found to apply to almost every form of the Polar type of the instrument.

Various attempts to classify the different forms of Planimeters, according to their mechanical action or some other standard of comparison, have been made, but have all proved unsatisfactory, owing to the fact that in many instances a given instrument while placed in one class on account of its possessing certain characteristics common to that class, may with equal reason be included in some other class, one or more of whose characteristics it may also possess.

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For this reason, the only actual and logical basis of classification or comparison is really one of relative degree of accuracy attainable by any given instrument and the range of operation of which the particular instrument may be capable.

It has been already stated in the introductory chapter that a knowledge of the theory of the design or operation of the Planimeter is not an essential either to an ability to use it or to apply it in the solution of the many practical problems for which its peculiar properties so eminently fit it.

It will readily be seen from the following discussion that what we shall term the "*General Principle*" of the Polar Planimeter is very simple, and can be stated in a very few words, while at the same time clearly expressing the fundamental theoretical considerations involved not only in the design and operation of the instrument, but also in all of its application to special conditions.

In fact, the first requisite for the intelligent successful use of the Planimeter and an ability to apply its principles to every form of practical application of which the instrument is capable, lies in a clear understanding of this "*General Principle*" rather than in a knowledge of how to mathematically demonstrate the truth of that Principle.

For this reason, while the mathematical proof of the truth of the "General Principle of the Planimeter" will be given as being necessary and proper to a clear and full theoretical treatment of the instrument, further mathematical discussion will be limited to showing how this General Principle can be utilized and adapted to various conditions so as to make the Planimeter the efficient mechanical aid to the Engineer which it is so capable of being.

The theory of the Polar Planimeter has been demonstrated in a variety of ways by writers on the subject and by others who have made a study of the instrument.

In some of these demonstrations use has been made of the Calcubus and other methods of the higher mathematics, while in others the desired results have been reached without going beyond the more simple of mathematical operations. While perhaps the use of the Calcubus would shorten the demonstrations, and in some cases render the mathematical treatment clearer, it is best for our purpose that the discussion be confined to the more ordinary processes of mathematical computation.

Symbols and Notation.

In order to make all demonstrations and discussions clearer and the mathematical treatment of the theory and applications of the Instrument connected and uniform throughout, certain letters and symbols will be assigned to represent the different parts or dimensions of the Planimeter, as well as the factors, constants and other values entering into either the descriptions of the instrument and its operations and applications or the mathematical discussion connected with such descriptions, so that when any such letter or symbol is employed or occurs in any portion of the subsequent treatise it will be at once understood as representing the quantity or value thus assigned it.

In the following demonstration of the general theory of the Polar Planimeter and the relation existing between its various parts, and in fact throughout all the subsequent discussion of the instrument and its practical applications, the different parts and dimensions of the Planimeter will then be denoted by the following letters, which are also used in all the diagrams and drawings used to illustrate the descriptions—it being

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remembered that all dimensions are given in millimetres and that the unit of graduation of the tracer arm is one-half millimetre.

Referring then to Fig. 4, Plate XII, which represents the principal parts of the Polar Planimeter, these parts and their dimensions will be designated as follows:

p = PF = Length of Polar Arm.
f = FW = Distance between Wheel and Carriage Pivot.
L = OT = Total length of Tracer Arm.
b = VW = Distance between Carriage Vernier Zero and Wheel.
s = OV = Setting.
t = FT = OT − (OV + VF) = Length of Tracer Arm.
r = Reading of Wheel = No. of Vernier Units recorded for tracing any area A with any given value of s or t.
c = Circumference of Wheel W.
x = Value of Relative Vernier Unit for any given operation.
y = Value of Corresponding Actual Vernier Unit.
R = Radius of Constant Circle.
A = Area traced with length of Tracer Arm t and which gives reading r for that tracing.
C = Constant for inside position of Pole.

Instrument Constants.

It will be readily seen from the description of the instrument given in Chapter II, and from the diagram, that certain of the quantities or dimensions thus designated are constant for any given instruments, while others vary and may be given any desired value.

Those dimensions which have a constant value for any given instrument are termed the "Constants" for that instrument or "Instrument Constants," and should be determined with great accuracy.

The "Instrument Constants" for the particular Planimeter we are using have been determined and are as follows:

p = 154.0	L = 208.0
f = 23.0	C = 61.24
b = 7.0	

all values being expressed in millimetres.

Of these values p, f, b and L are found by direct measurement of the parts themselves, while the value of c or the circumference of the Wheel W, which is evidently also a Constant for any given instrument, is found by calculation in the following manner:

Having very accurately drawn any plane figure having a known area, such for example as a square or circle having a known area of say 10,000 Sq. Millimetres, we carefully trace that area with any Setting whatever and take the Reading of the Wheel due to tracing that area; this Reading is the value r.

Let us suppose for example that having traced the known area of 10,000 Sq. Millimetres with a setting of say 29.4 the resulting Reading of the Wheel is say 1,000.

Since the graduation of the Tracer Arm is in one-half millimetres the distance OV represented by the Setting will evidently be $\frac{29.4}{2}=14.7$ millimetres and from Fig, 4, Plate XII.

$$t = FT = OT - OF = L - \left(\frac{s}{2} + b + f\right) \text{ .....(1)}$$

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But by measurement we have for this particular instrument b = 7.0, f = 23.0 and L = 208.0

Hence

$t = 208.0 - (14.7 + 7.0 + 23.0) = 208.0 - 44.7 = 163.3$ m/ms.

It will be demonstrated later that

$$c = \frac{A}{r \times t}$$

Hence substituting our known values of A, r and t we have as the desired circumference of wheel

$$c = \frac{10000}{1000 \times 163.3} = 61.24 \text{ millimetres}$$

In the case of the Coradi Compensating Planimeter and one or two other forms of the instrument the Zero of the Tracer Arm graduation is at the Tracer end of the arm and in such a position that the length of Tracer Arm for any operation or the value of t is always the same as the Setting divided by 2 which much simplifies this and other calculations.

As a full description of the Polar Planimeter and all its constituent parts has been given in detail in Chapter II no further description of any such part when referred to in the following demonstrations will be necessary; and when any letters or symbols referring to the instrument, its details or factors are used it will be considered that the list of such letters and symbols just given will have been sufficient to admit of the quantity thus referred to being at once recognized.

General Principle of the Polar Planimeter.

What we have referred to as the "*General Principle of the Polar Planimeter*" may be stated as being that "*The area traced by the Tracer during any operation is equal to the length of the Tracer Arm multiplied by the distance recorded by the Wheel for that tracing.*"

General Equation of the Polar Planimeter.

The mathematical statement of this General Principle is what we shall term the "*General Equation of the Polar Planimeter*" and is

$A = c \times r \times t$ ..... (1)

in which

A = Area of figures traced

c = Circumference of Integrating Wheel

r = Revolutions of the Wheel for that tracing

t = Length of Tracer Arm of the Planimeter which is always taken as being the distance between the Carriage Pivot F and the Tracer T.

The demonstration of the truth of this General Principle will now be given as concisely as is consistent with a clear understanding of the subject, after which the application of the principle as expressed mathematically in the General Equation of the Polar Planimeter will be fully discussed, the relation of the various essential parts of the instrument to each other and to the operation of the Planimeter described, and the derivation and significance of the factors with the methods for obtaining and applying their values fully described.

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Analysis of Movements of Integrating Wheel.

As the first step in the analysis of the theory governing the construction and operation of the Polar Planimeter it will be necessary to determine the effect on the wheel W of any motion of the Tracer T.

It is readily seen from the construction of the instrument that any motion of T must be either a motion of revolution of T about F as a center with a corresponding variable value of α, a motion of T about P as a center with a fixed value of α, or a combination of the two motions.

Thus, in Fig. 1 of Plate XII. in tracing the area of the figure shown, the motion of the Tracer while tracing the element of its path T'T may be considered as being composed of two component motions, T'S and ST. Of these components T'S is described by a motion of T about P as a center with a constant value of α, while ST is formed by a motion of T directly toward the Pole P with a corresponding variable value of α.

It is evident that each of these component motions will have an effect on the Wheel W, the kind and extent of the motion being dependent on the direction and length of the element T'T and hence on the value of the angle α.

In tracing any figure such as ABT'T it is seen that when the periphery of the entire figure has been passed over by the Tracer, the Tracer having returned to the point of beginning, that during the tracing the Tracer has moved just as much towards P as it has moved away from it and hence any revolutions of the Wheel due to movement *toward* P are neutralized by the same number of revolutions in an opposite direction due to movement *away* from P for that tracing:

This shows that the number of revolutions recorded by the Wheel during any given tracing of a closed figure are due entirely to the motion of T about P as a center.

In considering the path of the Wheel W due to motion imparted to the Wheel by motion of the Tracer T, it is seen that *when the direction of the path in any given case is at right angles to the axis of the Wheel*, the Wheel will move along that path by *rolling: When the direction of the path of the Wheel is in the direction of the axis of the Wheel*, the Wheel moves along the path by *slipping* and *when the direction of the Wheel's path is between these two directions* the Wheel moves along the path *partly rolling, partly slipping*, the amount of roll or slip in any given case being dependent entirely on the angle which the given path makes with the axis of the Wheel.

Denoting the angle made by any element of the Wheel's path with its axis by α and the length of the element by l we have for that element.

$$\text{Distance Rolled} = l \times \text{Sin } \alpha \text{ --------(1)}$$

$$\text{Distance Slipped} = l \times \text{Cos } \alpha \text{ --------(2)}$$

When $\alpha = 0^{\circ}$.

$$\text{Distance Rolled} = l \times 0 = 0.$$

and

$$\text{Distance Slipped} = l \times 1 = l,$$

When $\alpha = 90$ degrees,

$$\text{Distance Rolled} = l \times 1 = l.$$

and

$$\text{Distance Slipped} = l \times 0 = 0.$$

Revolutions of the Wheel and their Significance.

The component Rolling and Slipping Motions of Wheel due to movement of the Wheel along any element of its path being understood, the re-

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lation of these component motions to the operation of the instrument in tracing any given figure whose area is desired should now be considered.

In Fig. 2 of Plate XII let T'T be the component represented by TS of Fig. 1 of the motion of T while tracing an element of the periphery of a figure whose area is to be measured. Let Φ be the very small angle at the center subtended by T'T, O'O the arc of the Zero Circle, W'W the path of the Wheel during the movement T'T, and PE a perpendicular let fall on T'F' produced.

The dotted lines PW and PW' drawn from the Pole to the initial and final positions of the wheel for the given movement of T subtend an angle at P equal to the angle Φ.

The path W'W of the wheel is resolved into the two components W'S and SW ; SW representing the distance *rolled* by the wheel and W'S the distance *slipped* by the wheel during its movement from W' to W along its path.

Let U be the arc subtending the angle Φ at a distance of Unity from P.

Since W'W = Arc × Radius

we have

$$W'W = U \times WP \quad \text{.....(1)}$$

It has already been shown that the distance rolled by the Wheel for any element of its path is equal to the length of that element multiplied by the sine of the angle which that element makes with the axis of the Wheel: or from Eq. 1 of Pg

$$\text{Distance rolled} = l \times \text{Sin } \Phi \quad \text{.....(2)}$$

Hence

$$\text{Distance rolled} = SW = U \times WP \times \text{Cos. } W'WS \quad \text{....(3)}$$

Since Φ is very small the angle W'W'S can be considered as being equal to the angle PW'E.

But

$$W'E = W'P \times \text{Cos } PW'E$$
$$= W'P \times \text{Cos } W'WS \quad \text{.....(4)}$$

Hence substituting in Eq. 3

$$\text{Distance Rolled} = WS = U \times W'E \quad \text{.....(5)}$$

But

$$W'E = p \times \text{Cos } EF'P' - f = p \text{ Cos } \alpha - f \quad \text{.....(6)}$$

Hence

$$\text{Distance Rolled} = SW = U \times (p \text{ Cos } \alpha - f) \quad \text{.....(7)}$$

which is the distance rolled by the Wheel, while the Tracer is moved from T' to T along the periphery of the Fig. ABTT in Fig. 1.

The Zero Circle.

From the discussion just given of the Rolling and Slipping components of the travel of the wheel W, it is seen that there will evidently be some path or line along which the Wheel can be moved without producing any resulting rolling of the Wheel—the movement of the Wheel along this path being one of slipping only.

If we so adjust the positions of the two arms of the Planimeter that a line drawn from the Pole P to the Wheel W shall be at right angles to the Tracer Arm, and with the arms fixed in this position we cause the entire instrument to revolve about P as a center, it is seen that the Wheel W, the Joint F, and the Tracer T, will each describe arcs of concentric circles

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having P as a common center. It is also readily seen that since the axis of the Wheel is at every point of the circle thus traced by the point of tangency of the Wheel at right angles to the element of the circle at that point, there cannot be any rolling of the Wheel whatever so long as the arms retain this position with respect to each other. Of the three circles thus formed the circle described by the point of tangency of the Wheel is called the "Line of Slipping,"—that described by the Pivot Joint F is called the "Directrix," while the circle traced by the Tracer during this revolution is called the "*Zero Circle*," as shown in Fig. 2 of Plate III.

The "Zero Circle" can then be defined as being a line along which the Tracer can be moved without producing any rolling of the Wheel whatever.

Fig. 1 of Plate III shows a plan of the Planimeter when the Arms are in position to cause the Tracer to describe the Zero Circle, while Fig. 2 of the same Plate shows a lettered skeleton diagram of the instrument when in this position.

It is evident that α is always constant when the tracer is following an arc of the Zero Circle.

Referring to Fig. 2, just mentioned, it is readily shown that the Radius of the Zero Circle is

$$R = \sqrt{p^2 + 2\,ft + t^2}$$

while

$$\alpha = \text{Cos}^{-1}\,\frac{f}{p}$$

On examining the graduation of the wheel it will be seen that when the wheel revolves in the direction of a right-handed screw the revolution will be positive, while revolution in an opposite direction would be backward or negative.

When the Tracer moves about P as a center in the direction of the hands of a watch the *direction* of the tracing is *positive*, while motion in on opposite direction is *negative*.

Outside the Zero Circle, tracing in a *positive direction* gives a *positive rolling* of the Wheel while tracing in a *negative direction* gives a *negative rolling*.

Inside the Zero Circle, tracing in a *positive direction* gives a *negative rolling* of the Wheel, while tracing in a *negative direction* gives a *positive rolling*.

Relation of Roll of Wheel to Area Traced.

In Fig. 3 of Plate XII, let ABC be any given figure of whose area pq is an element. Let P, W and T be the Pole, Wheel and Tracer respectively of the Planimeter, and let the Tracer move across the width of the element at p, the width of the element being very small.

Let oo be an arc of the Zero Circle, the upper width of the element being coincident with it at the point q. Let U be the small angle at the Pole subtended by the width of the Element pq, while the parts of the Planimeter are designated by the letter assigned them.

Then

$$\text{Area pq} = \tfrac{1}{2}\,U\,(p^2 + t^2 + 2\,pt\,\text{Cos}\,\alpha) - \tfrac{1}{2}\,U\,(p^2 + t^2 + 2\,ft)$$
$$= Ut\,(p\,\text{Cos a} - f) \text{.....(1)}$$

But the area pq is the area included between the portion of the periphery of the figure traced, the arc of the Zero Circle, and the radii drawn from the Pole to the beginning and end of the line traced by T. We have

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already found that the distance which the Wheel would roll and record after tracing the width of the element at b is:

$$\text{Distance rolled} = U\,(p \operatorname{Cos} \alpha - f) \text{--------} (2)$$

Comparing Eqs. 1 and 2, it is seen that *the area of the element is equal to the distance rolled by the wheel for the given tracing multiplied by the length of the tracer arm t.*

As this is true for *any* element of area of the figure traced, it must be true of *every* element, and hence of the total area of the figure outside the arc of the zero circle.

It will require but little consideration to show that the instrument deals in like manner with that portion of the traced figure included *within* the Zero Circle, and that the rolling of the Wheel resulting from the tracing of the periphery of any given closed figure when multiplied by the length of the Tracer Arm gives at once the area of that figure.

If c is the circumference of the Wheel and r the number of revolutions made by the Wheel during any given tracing, it is evident that the distance rolled by the Wheel, which we have also referred to as the "roll" of the wheel, is $c \times r$. Hence, denoting the area of any figure by A, and the length of Tracer Arm by t, we have

$$A = c \times r \times t$$

which is what we have already termed the "General Equation of the Polar Planimeter," and from which is at once obtained the "General Principle" of the instrument.

Fundamental Equations.

From the "*General Equation of the Polar Planimeter*" just demonstrated, which is simply what we have termed the "*General Principle*" mathematically stated, we are able to at once determine the relation existing between the various parts of the instrument, and deduce the formulae by which the values of the different factors for the various practical applications are obtained.

As already given, this General Equation or Equation 1 is

$$A = c \times r \times t \text{----------} (1)$$

from which

$$C = \frac{A}{r \times t} \text{----------} (2)$$

$$r = \frac{A}{c \times t} \text{----------} (3)$$

$$t = \frac{A}{c \times r} \text{----------} (4)$$

$$r \times t = \frac{A}{c} \text{----------} (5)$$

Since the above five Equations show the relation of the various parts of the Planimeter to each other, and the relation of each to the operation of the instrument, they may be termed the "*Fundamental Equations*" of the Polar Planimeter, and will now be discussed.

Range of the Instrument.

In Eq. 5 it is evident that c is a Constant for any given Instrument, and A is a Constant for any given operation.

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For the particular instrument we are using we have already found—(Pg. 31)—C = 61.24 m/ms. Assuming A = 10000 Square Millimetres, and substituting in Eq. 5. we have

$$r \times t = \frac{10000}{61.24} = 163.292 \ldots\ldots\ldots\ldots (6)$$

This value of r × t is evidently a Constant for this particular instrument for the given value of A, which we have taken as 10000 Sq. m/ms.

Eq. 6 shows also that since r × t is a Constant for any given instrument and for any given area, r is dependent on t for its value, and any change in t produces a corresponding change in the value of r.

On examining the graduation of the Tracer Arm of our Planimeter, it will be seen that the minimum value which the construction of the instrument allows being given to t for the instrument is about 60 m/ms, while the maximum value is about 180 m/ms.

Substituting these values respectively in Eq. 6 and reducing, we have as the resulting No. of Vernier Units

$$r = \frac{163.292}{t} = \frac{163.292}{60} = 2721.5 \ldots\ldots\ldots (7)$$

and

$$r = \frac{163292}{t} = \frac{163292}{180} = 907.1 \ldots\ldots\ldots\ldots (8)$$

These Equations show that by changing the length of the Tracer Arm we can make the given Planimeter record any desired number of Vernier Units between 2721.5 and 907.1 Vernier Units while tracing an actual area of 10000 Sq. Millimetres; or in other words, that the *range* of the given Planimeter for an area of 10000 Sq. Millimetres is from 907.1 to 2721.5 Vernier Units. Since 10000 Sq. Millimetres equals 15.5 Sq. Inches the range of the instrument for 1 Sq. inch would evidently be from $\frac{2721.5}{15.5} = 175.55$ to $\frac{907.1}{15.5} = 58.52$ Vernier Units.

The reason why, both in the calculations of the Tables and in all or most of the mathematical demonstrations given throughout, the value usually assigned to A in any given operation is 10000 Sq. Millimetres, or its equal 15.5 Sq. inches instead of 1 Sq. inch is already stated in the "EXPLANATION OF TABLES," but will be explained in fuller detail later in the discussion.

It should be remembered that r is the distance rolled and recorded by the Wheel for any given tracing or operation, and can be expressed either as *the number of complete revolutions and fractions of a revolution*, or as the *number of Vernier Units* recorded, since the circumference of the Wheel is divided into 1000 equal parts or Vernier Units. In any event, it is always the "*Reading*" of the Planimeter for the given tracing or operation, and the change from Revolutions and fractions to Vernier Units is made by simply moving the position of the decimal point.

SETTINGS—THEIR DETERMINATION AND USE.

Having just shown the relation existing between the values of r and t for our particular instrument and the maximum and minimum values attainable for r for tracing an actual area of 10,000 Sq. Millimetres, we are able to explain the method by which we determine the value that

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must be given to t in order to obtain any desired value of "r" for the same area; in other words to find the length of Tracer Arm necessary to cause the Wheel to record any desired number of revolutions while tracing an area of 10,000 Sq. Millimetres.

Eq. 4 of our Fundamental Equations is as already given

$$t = \frac{A}{c \times r} \quad \text{------ (4)}$$

Substituting the value of c already formed for our particular instrument and making A = 10,000 Sq. m/ms we have

$$t = \frac{A}{c \times r} = \frac{10{,}000}{61.24 \times r} = \frac{163.292}{r} \quad \text{-------- (5)}$$

Since r is the number of revolutions which we desire the Wheel to record while tracing the given area of 10,000 Sq. Millimetres, it is evident from Eq. 5 that the required length of Tracer Arm is obtained by dividing the Constant 163.292 by the desired reading or value of "r."

The meaning of the term Setting together with the method of adjusting the Planimeter to any given Setting has been fully described in the preceding chapter and a glance at Fig. 1 of Plate III will at once show the relation existing between any given Setting and the corresponding Length of Tracer Arm for any given instrument.

To determine this relation for the particular Planimeter we are using we make use of the skeleton diagram of the instrument shown by Fig. 4 of Plate XII, which shows the essential parts properly lettered, together with the instrument Constants and other dimensions determined either in the manner already explained for finding those Constants, or by direct measurement of the instrument in question.

It is to be remembered that the unit of graduation of the Tracer Arm, being a half Millimetre the value of any Setting is expressed in that unit and hence the actual distance is always one-half of the distance as expressed by the value given for the Setting, or in other words the actual distance S is $\frac{\text{Setting.}}{2}$

Referring to the diagram it is evident that

$$t = FT = OT - (OV + VF)$$

But for this instrument

$$OT = 208 \text{ Millimetres}$$

$$OV = S$$

$$VF = b + f = 7 + 23 = 30 \text{ Millimetres}$$

Hence

$$t = 208 - (S + 30) = 178 - S$$

from which

$$S = \frac{\text{Setting}}{2} = 178 - t \quad \text{------ (6)}$$

Eq. 6 shows that after having found by Eq. 5 what value of t is required in order that the Planimeter shall record the desired number of Revolutions while tracing an area of 10,000 Sq. Millimeters, by substituting this value of t expressed in Millimetres in Eq. 6 and solving we at once obtain the Setting to which the Planimeter must be adjusted in order that the tracer arm shall have the length required.

By substituting for t in Eq. 6 its value as expressed in Eq. 5 we have

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as the general Equation for finding the Setting required to give any desired value of r for the area of 10.000 Sq. Millimetres,

$$\text{Setting} = (178 - \frac{163292}{r}) \times 2$$

or

$$\text{Setting} = 356.6 - \frac{326.584}{r} \quad \ldots\ldots (7)$$

It is evident that the Setting necessary for any instrument to cause it to record any desired number of Vernier Units for any given area can be found by trial by repeated tracings of the given area with different lengths of tracer arm, but except as a check in any given case the method by calculation as just described is much quicker and easier of application.

To illustrate the application of the formulæ just deduced let us suppose that we desire to find what Setting our given Planimeter must be adjusted to in order that the instrument should record say 1,550 Vernier Units when tracing an actual area of 10,000 Sq. Millimeters.

Eq. 7 for this particular Planimeter is as just shown

$$\text{Setting} = 356.6 - \frac{326.584}{r} \quad \ldots\ldots\ldots (7)$$

Since the desired value of r is 1550 Vernier Units or, when expressed in Revolutions of the Integrating Wheel, 1.55 Rovolutions, we substitute this value of r in Eq. 7 which gives

$$\text{Setting} = 356.6 - \frac{326.584}{1.55} = 356.6 - 210.61 = 145.9$$

which is the Setting desired.

The reason why the actual area of 10,000 Sq. Millimetres or its equal 15.5 Sq. Ins., is always taken in this calculation and in fact in the calculation of all the factors in the Tables is that not only does it simplify the calculation, but it is usually one of the constant areas found on the Test Plate accompanying most Planimeters and hence makes the checking of the calculation a very simple matter.

Thus, in the example just given, having obtained the value of the desired Setting in the manner shown we at once adjust the Planimeter to 145.9, the Setting in question, and trace the given area (10,000 Sq. Millimetres) with the aid of the test plate. In this way the accuracy of the calculation is at once tested.

The Cols. marked "Area" and "Reading" under the heading "Test Plate" in all the Tables in connection with the values given in the "Relative" and "Actual Vernier Units" columns evidently give at once a very accurate test, both of the accuracy of the Setting for any given scale and of the adjustment and condition of the Planimeter and these should be used in connection with the Test Plate area of 10,000 Sq. Millimetres to check the accuracy of the Planimeter before each use of the Planimeter.

On examining the diagram given on Plate II, it is readily seen that the curve BB is simply the graphical representation of Eq. 7 for the various values of r for an area of 10,000 Sq. Millimetres between the limits of the given instrument; the curve being the various values of the Settings and having for its ordinates the values of r while the abcissa are corresponding values of S.

The determination for any given Planimeter of the Setting necessary to cause the instrument to record a certain number of desired Vernier

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Units, when tracing a given area is greatly facilitated by the use of a diagram similar to that given on Plate II.

As this diagram can by applying a proper connection be used for any Planimeter similar to the one we are discussing the method of preparing the diagram will be described at this point, and its manner of use described. The size of the diagram having been decided upon, it is ruled in squares in the manner shown and the range of the instrument having been determined, the values of r due to tracing a constant area of 10,000 Sq. Millimetres between the maximum and minimum values of the length tracer arm are lettered in a vertical column at the ends as shown.

The upper base of the diagram is taken as representing to some scale of equal parts, the various divisions and subdivisions of the Tracer Arm graduation and lettered or numbered accordingly.

We have seen that the Setting necessary to give any desired Reading or value of r for the particular Planimeter we are using is given by Eq. 7 just demonstrated and is

$$\text{Setting} = 356.6 - \frac{326.584}{r} \quad \text{.....} \quad (7)$$

If now we take the various values of r between the limits of the range of our instrument and substitute them successively in Eq. 7—solving each such resulting Equation—we can readily plot the resulting values of the respective Settings on the diagram— marking each such location by a dot —and by connecting these located dots by a curved line we shall have the curve BB.

It is evident that this curve is the curve of Settings for the given Planimeter plotted to co-ordinates in which any ordinate to the curve is a number of revolutions and its corresponding abcissa is the distance represented by the Setting (or the distance S) necessary in order that the wheel shall record that number of revolutions when tracing an area of 10,000 Sq. m/ms.

In the diagram the curve BB is for use with the Planimeter we are using while the curve AA is the curve of Settings for those Planimeters like the Compensating Planimeter which has the Zero of its Tracer Arm graduation at or near the Tracer. The upper base is numbered for use with the curve BB while the lower base is for use with the curve AA.

In plotting if the curve of the even numbered revolutions—1,000, 1,100, 1.200, &c are calculated and plotted and the curve constructed from their values the intermediate values will be found quite accurate enough for any but the most accurate operations.

Vernier Units.

In the discussion of the "*Range of the instrument*" it was shown that while the Planimeter could be made to record different number of Vernier Units for the same area by changing the length of the Tracer Arm, there was a maximum and minimum number of Vernier Units which could thus be obtained for any given area and any given instrument.

In the case of the particular Planimeter used, it was shown that for that instrument and for an actual area of 10,000 Sq. Millimetres, the instrument was capable of recording any desired number of Vernier Units between 907 and 2,721.

It very often happens that the conditions of some problem may demand that the instrument should deal with values greater or less than the limits thus found. This is very readily accomplished in effect by giving an *assigned* value to the *actual* Vernier Unit so as to cause *one recorded*

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actual unit to represent as many *assigned units* as the conditions of the problem may require.

Thus, suppose, with the instrument having the range already determined, we wish it to record say 3,200 Vernier Units for the given area.

As 2,721 is the maximum number of Vernier Units which this particular instrument is capable of recording for the given area, we can in effect obtain the desired result by assigning a value of 2 to each actual Unit recorded, and then making the Planimeter record 1,600 Actual Units for the given area.

Since one actual recorded Vernier Unit has an assigned or relative value of 2 for the given operation, by multiplying the number of *actual* Vernier Units recorded for the given operation by 2, the assigned or relative value, the result is exactly the same as though we had been able to make the instrument record the desired 3,200 units.

Hence in every problem the Vernier Units recorded during the operation of the Planimeter may have, and usually do have, two distinct values: one value the "*Actual*" or absolute value, and the other the assigned or "*Relative*" value dependent upon the special conditions of the given problem.

Thus, for example, in the case of the measurement of any area with the Planimeter, such for instance as a map drawn to some given scale, the number of recorded Vernier Units when multiplied by the "Actual" Vernier Unit value would give the *actual* area of the map traced in actual sq. ins. or other unit, while by multiplying the same by the assigned or "*Relative Vernier Unit*" value we would obtain the area in sq. feet, sq. miles, acres, or any other unit demanded by the scale of the map and other conditions of the problem.

An additional reason for the frequent use of the Relative Vernier Unit is that in most measurements with the Planimeter it is desirable that the length of tracer arm should be as great as possible consistent with other conditions, and this the employment of the Relative units will in most cases admit of.

The significance and use of the Relative Unit will be more clearly understood from its frequent use in the following descriptions of the practical applications of the Planimeter.

Constants—Their Deriviation and Use.

While the demonstrations and descriptions thus far given cover all the factors involved in the ordinary methods of Planimeter measurements, in a succeeding chapter is given a method by which the areas of figures too large to be measured by the ordinary methods can be accurately determined.

It will be seen from the description of the method referred to, that having placed the Pole of the instrument on the inside of the large figure to be measured and traced the given figure, the resulting Reading must be added algebraically to a certain "Constant" before the desired area can be obtained.

The derivation and significance of these Constants are very easily understood in the light of the previous discussion of the Zero Circle and its relation to the theory of the Planimeter.

In the discussion referred to it was shown that the distance rolled by the wheel or the "roll" of the wheel when multiplied by the length of the Tracer Arm gave the area of that portion of the figure included between the path of the Tracer, the arc of the Zero Circle, and the radii drawn to the initial and final positions of the Tracer.

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It is evident from this that if the area of the figure to be measured is a large one, and we place the Pole near the center of the given area and trace its periphery, the resulting Reading of the Planimeter, when multiplied by the length of the Tracer Arm, will give the total area of the figure traced, *minus* the area of the Zero Circle. In other words, the instrument will evidently measure the difference in area between the area of the figure thus traced and the area of the Zero Circle, and hence the area of the given figure will be the algebraic sum of the area recorded by the Wheel and the area of the Zero Circle.

This is true regardless of either the relative or actual sizes of the two areas.

The Radius of the Zero Circle has already been shown to be

$$R = \sqrt{p^2 + 2\,ft + t^2} \quad \text{----------(1)}$$

and its Area is

$$\eta R^2 = \eta (p^2 + 2\,ft + t^2) \quad \text{----------(2)}$$

In finding the *actual* area of any given figure by this method the constant used would evidently have to be the area of the Zero Circle, expressed as the number of Vernier Units which would be recorded by the Planimeter if the Constant Circle were traced with the given Setting for the operation in which it occurs.

In other words, the Constant for the measurement of any area drawn to a given scale is evidently the difference between the number of Vernier Units actually recorded for the given area and the number of Vernier Units which would be recorded could the entire area have been measured by the Planimeter with the Pole on the outside of the given figure.

It is readily seen that the actual area of the Zero Circle must change for any change in t, since p and f in the expression $p^2 + 2$ ft. $+ t^2$ are constant for any given instrument and t is variable, and hence there must be a corresponding value of the "Constant" for every given "Setting."

In the description given in Chap. IV of the method of measurement of large areas with the Pole on the interior of the given area, the significance, derivation and use of the "Constant" is so fully described that further discussion of the subject at this point is unnecessary, although the theory and application of the Constant should be easily understood from what has already been said here on the subject.

In fact, in all the following descriptions of the use of the Planimeter in all its many practical applications, the relation of the theory of the particular operations described is made clear, thus supplementing the discussion given here of the theoretical considerations involved.

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CHAPTER IV.

Measurement of Plane Areas.

I. MEASUREMENTS IN GENERAL.

METHODS OF MEASUREMENT.

There are two methods by which the Area of any plane figure may be accurately measured by the Polar Planimeter, the method selected for any given case being dependent on the size of the given Area and the capacity of the Planimeter used. The first of these two methods which we are about to describe is given under the heading "MEASUREMENT OF SMALL AREAS," and is the method invariably employed when the size and shape of the figure to be measured is such as to allow of *every* point of its perimeter being reached by the Tracer without moving or shifting the position of the Pole during the tracing, and having the Pole outside of the given area.

The second method of measurements is described under the heading "MEASUREMENT OF LARGE AREAS," and is used when the area of the given figure is too large to be measured by the first method, or when the size or shape of the figure to be traced is such as not to allow of *every* point on its perimeter being reached by the Tracer when tracing the figure without moving the Pole—the Pole being on the outside of the given figure.

These two methods are distinguished and usually designated as "Measuring with Exterior Pole" and "Measuring with Interior Pole," respectively, and, as has been above intimated, the first method of measurement or "Measuring with Exterior Pole" is the method always selected when the conditions as specified above admit of its employment.

The measurement of the area of plane figures will now be taken up, the use of the Planimeter in both methods of measurement described in detail, and the conditions, both theoretical and practical, on which the maximum degree of accuracy in operating and results depend, will be explained in as clear a manner as possible.

As the actual operation of the Polar Planimeter in all its many particular applications is exactly the same as in the single one of obtaining the area of a plane figure, the following description of the manner of operating and the directions given for obtaining the highest possible degree of accuracy and efficiency will apply with equal force to its use in *every* application, and should be clearly understood and carefully followed in every case.

Keeping in mind what we have elsewhere termed the "GENERAL EQUATION" or "FUNDAMENTAL PRINCIPLE" of the Polar Planimeter which was deduced in the demonstration of the General Theory of the Planimeter given in Chapter III, the theoretical conditions involved in each and every operation of the instrument can be clearly seen. It will also prove how essential to accurate results it is that the directions given be implicitly followed, as well as give an intelligent understanding of the working of the Planimeter, and an appreciation of the wonderful accuracy

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and practical value of the Planimeter in every form of Engineering and Scientific computation.

The measurement of the area of any plane figure by means of the Polar Planimeter is a very simple operation, and when carefully performed with a Planimeter in good condition and adjustment, and with the conditions for accuracy properly observed and complied with, will give results with a degree of accuracy not attainable by any other method of measurement.

This is especially true in the case of figures having irregular bounding lines, or when a portion or all of the periphery of the given area is composed of curved or broken lines, since the shape or nature of the outlines of the figure have no influence whatever on the accuracy of the measurement.

Examination and Adjustment of the Planimeter.

Before beginning any operation with the Planimeter, and especially if the Planimeter has not been in very recent use, the condition of the instrument and its adjustments should be carefully examined, and with particular reference to the conditions mentioned in Chapter II.

After a general examination of the instrument for apparent injury or defect due to accident, such as a bent needle or arm, rust or dent on the rim of the Integrating Wheel, binding of the carriage pivot, etc., the Planimeter should be tested for sources of error not discoverable by ordinary inspection. It is to the latter that we must look for cause and correction in almost every case when the Planimeter fails to give a result with its customary degree of accuracy, and the instruments must be subjected to a series of tests to determine the source and extent of the inaccuracy and the means of correction.

To test the accuracy of the Planimeter the Test Plate is fixed on the paper, a starting point accurately marked, and the Tracer being placed in one of the small holes already described, the Test Plate and Tracer is made to revolve a certain number of even times about the needle point fixed in the end of the Test Plate as a center. The Reading of the counting and Integrating Wheels being taken at both the beginning and end of these revolutions, the difference of these two Readings is evidently the Reading of the Planimeter for the number of Revolutions of the Tracer thus made. If at the end of these revolutions the Tracer is caused to make an equal number of revolutions in the opposite directions from the first, and the Reading of the Instrument due to the reverse revolutions is equal to the Reading due to the first or direct revolutions, the assumption is that the Planimeter is in good adjustment.

If, however, these Readings differ, or what is the same thing, if the reverse tracing of any figure does not give the same Reading as did the direct tracing of the same figure, the instrument is evidently either not in adjustment, or else one or more of what we have termed the "conditions for accuracy," have not been complied with.

This discrepancy in the readings thus obtained may then be due to irregular slipping of the Integrating Wheel, lost motion, irregularities in the surface of the paper on which the Wheel rolls, non-parallelism of Wheel Axis with Tracer Arm, or lack of sensitiveness in the Wheel Axis bearings due to causes already discussed in Chapter II.

If we measure any given area which is placed first on the *outside* of the Constant Circle and then on the *inside* of the Constant Circle, and the Readings for both positions of the same area are the same, it proves

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that the defect in or lack of accuracy is not due to any displacement of the axis of the Wheel from its proper position, and that the axis of the Wheel is parallel to both surface and Tracer Arm.

The unfavorable effect of the surface of the paper on which the Integrating Wheel moves is easily determined by pinning to the drawing board a piece of paper, having what experience has proved to be a favorable surface for the purpose, and of such shape and size as to contain within its area the entire path of the Wheel during the measurements in question.

Should the axis of the Wheel be too loose or shake in its bearings, it can easily be detected by forcing the Tracer Needle into the paper, and then after having fastened the binding screw, trying to shake the axis in its bearings by moving the other end of the Tracer Arm laterally with the fingers.

Lack of sensitiveness of the Wheel Axis with its cause and method of correction has already been treated in a previous chapter and need not be repeated here except to say, that when the requisite degree of sensitiveness has not been restored by any of the methods described, it is safe to assume that the defect is due to dullness, or is the result of injury by shock or accident to the wheel axis or its bearings, and the only remaining remedy is to place the instrument at once in the hands of a competent maker.

All errors or inaccuracies in results of Planimeter measurements due to the non-parallelism of Wheel Axis with Tracer Arm, or to imperfect working of the Integrating Wheel or its axis from dirt or looseness in its bearings, are, however, easily discoverable by the testing methods just described, and disappear when the axis and its bearings are properly adjusted. These adjustments are made by means of the bearing and set screws in the Carriage, and by them the Wheel and its attachments can be adjusted with the greatest nicety, and with mathematical and mechanical precision.

II. Measurement of Small Areas.

As previously stated, what is meant by "Small Areas" in this connection are areas of such shape and size as to permit of every point of their periphery being reached by the Tracer without change of position of the Pole and when the Pole is entirely outside of the figure to be measured.

The accuracy of all measurements made with the Planimeter depends upon the condition of the Planimeter, upon the accuracy and care with which the measurement is made and upon the strict compliance with those details of operating which theory and experience have shown to be conditions for accuracy. Many of these conditions have been already described or referred to in previous pages and others will be given as their application becomes evident in the following descriptions. The insistance on the strict observation of these conditions will be seen to be justified when their importance is more fully understood and the effect of this observance on the accuracy of the practical working of the Planimeter is shown.

After having examined the condition of the Planimeter and made any adjustments found necessary after testing the instrument in the manner just described, the method of measurement of the given area is as follows:

Adjustment of Drawing and Instrument.

The plan or map on which is the figure whose area is to be measured is smoothly laid on and attached to the drawing board or table, the surface

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of which should be level and free from irregularities: especial care being taken that the surface should be smooth and without uneveness due to warping. If the paper on which the plan or map is drawn is in good condition and lies smoothly on the board without wrinkles or tear it can be fastened to the board with the ordinary thumbtacks. If however the paper is torn, wrinkled, or badly creased so that it does not present an even surface or lay flat on the board it must be stretched and mounted with paste on the board, in the same manner as when water colors are to be applied with a brush.

In the case of old and valuable maps or drawings which cannot be mounted without injury to the map itself, they should be tacked down as smoothly as possible to the board and a sheet of paper or card board tacked on that portion of it on which the Wheel will move during the tracing of the given figure as has already been described. The size and position of the paper thus used can be at once determined by observing the path of the Wheel while making a preliminary tracing of the figure.

The map or drawing having been fixed to the drawing board with its surface as true and even as its condition will admit of, the Carriage Venier V is adjusted to the SETTING as given in the Tables for the scale to which the figure is drawn, and the Tracer Arm firmly clamped to the Carriage by the proper screws. The Tracer is then placed near the center of the given figure and the Pole placed in its proper or most favorable position on the paper with respect to the area to be measured. The proper or "most favorable" position of the Pole with reference to any given figure is easily determined by the Rule already given as well as the selection of the point for beginning the measurement. That the Pole be placed in this position is a matter of considerable importance in obtaining the highest degree of accuracy of results, and to facilitate the finding of this position and illustrate the application of the Rule, Figs. Nos. 2, 3, 4 and 5 are given on Plate V which show the most favorable positions as well as the proper direction of the tracing and places for beginning of the four principal forms of Planimeters with respect to the given figures whose areas are to be determined.

TRACING THE FIGURE.

The Planimeter having been placed in position and a repreliminary tracing of the figure to be measured having been roughly made to see that every point on its perimeter can be reached by the Tracer, the Tracer Rest is adjusted to the proper height and the Tracer brought to the point of beginning. With the Tracer at the point of beginning and before commencing the tracing the Reading of the instrument is taken which, as we have already seen, is the Reading of the Counting Wheel O and the graduated drum of the Integrating Wheel W. Then carefully and in the directions of the hands of a watch the Tracer is conducted along the outlines of the given figure by taking the Tracer Rest E (Plate I, Fig. 3) between the thumb and first finger of the right hand and guiding the Tracing Needle slowly but steadily and with a uniform motion along the center of the bounding lines, steadying the motion by resting the palm and the other fingers of hand upon the paper, until the Tracing Needle has traced the entire circuit of the given figure and arrived again at the point of beginning. A second Reading of the Planimeter is then taken and the difference between the first and second Readings, obtained by subtracting the Reading taken before the tracing from the Reading taken at the completion of the tracing, is evidently the Reading due to the tracing

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of the given figure or the "Reading for the given area." This Reading which, as has already been explained in Chap. III, is the number of "*Wheel Units*" or "*Vernier Units*" recorded by the Planimeter while tracing the given figure, when multiplied by the value of the Relative Vernier Unit given in the Table for the scale to which the figure is drawn gives the desired area of the given figure traced. The description just given of the method of measuring the area of any plane figure will perhaps be more clearly understood by means of the following practical example. It will also illustrate the simplicity and ease with which the area of any figure regardless of its shape or the nature or irregularity of its periphery can be at once obtained with the Polar Planimeter and with a facility and accuracy unequalled by any other known method.

EXAMPLE I.

Example of Measurement.

Let us suppose the figure whose area is desired to be drawn to a scale of 1 inch = 30 feet, and we desire to measure the area with the Polar Planimeter—the area to be expressed in Sq. Ft.

Looking in Table 1 on Plate I we find the Setting of the scale 1″ = 30′ to be 121.8. Unscrewing the screw *L* (P. Plate 1, Fig. 1), we slide the Tracer Arm *RS* through the sleeves of the carriage until the first small division beyond the division line marked 12 is made to coincide with the Zero line of the Carriage Vernier *V*. Clamping the screw *L* the arm is moved by the slow motion screw *M* until the small line 8 on the Carriage Vernier is made to coincide with the first line encountered by it on the Tracer Arm which in this case is the line 129. This being accomplished the Planimeter is now in adjustment for making the desired measurement.

The Planimeter is now placed in correct position with respect to the figure to be measured, and the Pole placed in the "most favorable position," as explained above, and the Tracer brought to the place of beginning or starting point determined upon. The Reading of the Instrument is then taken; let us suppose that the index of the Counting Wheel *O* points to or a little beyond the line marked 3, and that the graduated drum of the wheel W reads 324. The first Reading of the Planimeter is then 3324.

The Tracer is now carefully guided along the periphery of the given figure in the manner and with regard to the conditions already given until the Tracer having traced the complete boundary of the figure arrives again at the point of beginning. The Reading of the Planimeter is again taken, and let us suppose that the Counting Wheel now reads 7 and the wheel *W* reads 597, which makes the reading of the instrument 7597. If now we subtract the first Reading 3324, from the second or final Reading 7597, we have for the Reading for the given figure, 7597 — 3324 = 4273, which shows that the Integrating Wheel *W* has made 4 whole revolutions and 273 thousandths of a revolution while the Tracer has been tracing the periphery of the given figure; or which is the same thing, the wheel has recorded 4273 Vernier Units while tracing the given figure.

Looking now in Table 1 for the value of the Relative Vernier Unit we find that for the given scale, 1 in. = 30 ft., one Vernier Unit recorded by the wheel represents an area of 10 sq. ft., and as the wheel has recorded 4273 Vernier Units for the given figure the area of that figure must be 4273 × 10 = 42730 Sq. Ft., which is the required result.

If for purpose of checking the accuracy or for other reasons the *actual* area in square inches of the figure just measured should be desired it can

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of course be readily measured by setting the Planimeter to the Setting 145.3 which with Vernier Unit .01 is the Setting for the natural scale of 1 inch = 1 inch for the instrument used. The figure being again traced with the new Setting will then give a Reading which when multiplied by .01, the value of the Vernier Unit, will give the actual area in square inches.

To facilitate this operation and admit of the measurement of the given area in square inches without changing the Setting or having to make a second tracing of the figure, a set of factors have been calculated and placed in all the Tables giving what we have termed the *Actual Vernier Unit* for each of the given scales or settings. The value of the Actual Vernier Unit in any case is such that if we multiply the Reading for any figure traced by the value of the Actual Vernier Unit found in the proper column opposite the scale in question, the product will be the *actual* area of the given figure expressed in square inches. For example, in the example just given, the reading of the instrument for the given figure was, as we have seen, 4273 Wheel Units. Looking in the column headed Actual Vernier Units opposite the given scale, 1" = 30' we find the value of the Actual Unit to be .1111, which means that each Unit recorded represents an actual area of .1111 Sq. Ins. The actual Area of the entire figure will then be $4273 \times .01111 = 47.47$ Sq. Ins. Since in our example 1 Sq. In. = 900 Sq. Ft. the check is made by simply multiplying the total actual Area or 47.47 Sq. Ins. by 900, which gives the 42730 Sq. Ft. found by the first measurement.

In the example just given should the area be desired in Acres instead of Sq. Ft. the measurement would be made in exactly the same manner, except that the Planimeter would be set to 50.0, which is the Setting given in Table 4 for the Scale 1 in. = 30 ft., and the resultant Reading would be multiplied by .0003, which is the corresponding value of the Relative Vernier Unit for the scale in question.

When we consider that the area of any plane figure, regardless of the irregularity or nature of its periphery which may be curved or broken to any extent, can be measured by the simple operation described, and its area determined within a degree of accuracy of say 1/20 of 1 per cent. by the simple Polar Planimeter, we get some idea of the almost incalculable value of the instrument as an aid to professional or scientific work.

Formula for Area.

In the operation just described

Let A = Area of Figure

r_1 and r_2 the first and 2nd Readings respectively

x = value Actual Vernier Unit and y = Value Relative Vernier Unit.

Then

A in Sq. Ft. $= (r_2 - r_1) \times y$ --------(1)

and A (Actual Area) $= (r_2 - r_1) \times X$ --------(2)

It is evident from the description just given that the operation of measuring an area with the Polar Planimeter is a very simple one and singularly free from liability to error, since but one mathematical operation is involved—(the multiplication of the Reading by the value of the Vernier Unit) and that operation usually a mental one as the value of the Vernier Unit is usually a single simple number, often Unity. It is also evident that with an instrument in good adjustment and condition, the accuracy of the measurement is entirely dependent on the accuracy and fidelity with which the Tracer is made to follow the periphery of the given figure.

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Conditions for Accuracy.

Experience has shown that the accuracy of the Tracing is increased and the operation greatly facilitated by the observance of the following details during the operation of tracing the figure.

Whenever possible the tracing of any line should be watched or viewed in the direction of the line, as by observing this precaution the Tracer can be kept to the center of the line with greater accuracy, and any deviation from its true path is more readily detected by the eye.

At each angle of the figure, or when there is any decided change in the direction of the periphery, the finger should be placed on the top of the Tracer and the point of the Tracer pressed down into the paper at the given point, thus allowing the guiding hand to be rested and to assume the most convenient position for guiding the Tracer along the new direction without danger of moving the Tracer while making the change.

The motion of the Tracer along the periphery should be slow and uniform, and if from any cause the Tracer should deviate slightly at any point from its path by accident, it can be compensated for by causing an equal deviation to the *opposite* side of the path and for a length along the path equal to the length of the accidental deviation. A new tracing is, however, evidently preferable when a high degree of accuracy is desirable in the given operation.

While tracing any given figure, it should be noted whether the Counting Wheel *O* has passed its O mark during the operation. If it has done so, and its index points to a figure beyond O, the reading for the given figure must be obtained by subtracting the first reading from 10,000 and *adding* the second reading to the difference.

To avoid complication and minimize the possibility of error, the Integrating Wheel is often set to Zero before beginning the tracing, and often the entire instrument is set to read Zero by turning the Counting Wheel to Zero as well as the Integrating Wheel. The advantage of this procedure is, however, doubtful, except in the case of very long and continuous tracings in one operation, as the temptation is always present to turn the wheels with the finger—an operation to be rigidly excluded. If, however, the Zero of the Integrating Wheel is sufficiently near to Zero as to require little turning to bring it there, it can be so set without injury to the Planimeter by placing the Instrument in its proper position with the Tracer at the point of beginning and then causing the wheel to read Zero by slightly shifting the position of the pole, the Tracer being pressed down at the starting point during the shifting of the pole. This has the advantage of making the second Reading the reading for the given figure by the simple mental subtraction of the two readings of the Counting Wheel alone, and is often of advantage when performed in the manner specified.

Use of Guide or Template.

In many cases of measurement with the Planimeter, and especially in cases where the figure to be measured has an outline composed of long straight lines, the Tracer is guided along these lines by causing the edge of a flat thin straight-edge or ruler to coincide with the line to be traced and the side traced by carrying the needle along the edge of the ruler thus placed.

There is considerable difference of opinion as to the value of this method of tracing. M. Coradi, certainly an authority, condemns the practice as being a source of error rather than an aid to accuracy, arguing that owing to the more or less spring or bend of the Tracing Needle due

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to its being pressed against the edge of the Ruler, there will exist a constant error in the result of the measurement of any area whose periphery has been thus traced. He further supports his contention by deducing the error due to this "spring" of the needle in tracing a circle of 8 c/ms radius. Assuming the spring due to pressure of the needle against the guiding edge to amount to a deviation of the point of 0.02 m/ms from its normal position, he finds the error in the area of the circle in question due to this deviation to be 1/2000 = 10 Vernier Units, and remarks that "*when one thinks that it takes but the slightest effort to put the Tracer 0.1 m/n out of its true position, I believe that what has been stated has been fully verified.*"

The experience of the writer has been, however, that the use of a ruler as a guide to the Tracer not only greatly facilitates the tracing, but that repeated tests made by measuring a figure of known area, both with and without the aid of the guide, have in every case not only failed to detect any error due to spring of the needle, but have shown an increased accuracy in every case where guide was employed. Certain it is that the amount of deviation due to any such spring can be minimized and practically eliminated by giving a conical shape to the Tracer, by pressing the point of the Tracer against the guide with only sufficient force to cause it to keep in constant contact with the guide, and by the simple precaution of moving the guide sufficiently away from the line traced as to cause the deviated path of the needle to coincide with the center of the line traced.

The accuracy of the results obtained in measuring with the Planimeter volumes of continuous prismoids, discussed in a later chapter, which necessitates the continuous use of a template or guide, is also proof that when properly used and with the precautions given above properly taken, the use of a guide when tracing straight sided figures is conducive to *increased* rather than *decreased* accuracy in the measurement.

Accuracy of Results.

With the Planimeter as with the Transit, and in fact with most measuring instruments, almost any degree of accuracy in the result of the measurement of any given quantity may be obtained by the method of repetition. Instead of taking the Reading due to one tracing of the given figure, the tracer is made to trace the figure any desired number of times, and the total Reading of the instrument due to the number of tracings made, divided by the number of tracings, is taken as the true Reading.

Space will not be taken here to discuss the relative or absolute accuracy of Planimeter measurements, as the question of accuracy is taken up and fully discussed in Chap. XI. Suffice it to say that in the vast majority of cases the Polar Planimeter, by the simple tracing of a plane figure as just described, will give its area with a degree of accuracy entirely impossible by any other known means.

II. MEASUREMENT OF LARGE AREAS.

Adjustment of Drawing and Instrument.

Following our division of the methods of Measurement of Plane Areas we come to the Second Method or Method of Measuring Large Areas. This method, as stated at the beginning, is the one used when the extent or shape of the figure whose area is desired is such that every point on its periphery cannot be reached by the Tracer from one position of the Pole with the Pole on the outside of the given figure. As indicated by the

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term "Measurement with Interior Pole" commonly applied to designate this method, the Pole of the Planimeter is placed within the given figure to be measured, usually at or near the center of the figure, and the Tracer makes a complete revolution about the Pole as a center while tracing the periphery of the figure.

The operation of measuring the area of a plane figure by this method is quite as simple as that just described and is as follows:

Tracing the Figure.

The Setting for the scale to which the given figure is drawn being taken from the column of Settings opposite the given scale, the Planimeter is adjusted to that Setting and clamped. The Pole of the instrument is then placed at or near the center of the given figure, care being taken to see that every point on the perimeter of the figure can be reached by the Tracer from one position selected for the Pole, which can easily be determined by a rough preliminary tracing of the figure. A place of beginning is then selected and marked and the Tracer having been brought to this point the Reading of the Instrument is taken. Then carefully and in the manner described and with due attention paid to those details already given as requisite for accuracy, the Tracer is made to trace the periphery of the given figure in exactly the same manner as described for the tracing of the small area. When the Tracer has traced the entire circumference of the figure and arrived at the place of beginning, the Reading for the given area is obtained as in the first method by subtracting the first Reading from the second or final Reading. Unlike the first method however the area of the figure is *not* the product of the Reading of the given area by the value of the Vernier Unit for the given scale. The area recorded by the instrument during this operation is actually the difference in area between the area of the figure traced and the area of the Constant Circle, and hence to obtain the area of the figure, we must add to the Reading given the area of the Constant Circle for the scale of the figure expressed in Vernier Units and given in the Constant column of the Tables for that given scale. The sum of these two values will then be the area of the given figure expressed in Vernier Units and this sum multiplied by the value of one Vernier Unit given in the Table for the scale of the figure will then be the area required.

Movements of Integrating Wheel.

While tracing the periphery of the figure, in the manner described, if the Integrating Wheel be watched, it will be found that the direction of the rotation of the wheel may not be the same throughout the tracing. At some places of the periphery the tracing may cause a forward or positive rotation while at other places it may cause a backward or negative rotation and hence the final Reading will really be the difference betweeen the amount of the forward or positive rotations and the amount of the backward or negative rotations; or *taking into account the algebraic signs* of the two directions of rotation, the positive rotations being +, and the negative —, the final Reading will be the Algebraic sum of the two, and the proper sign being given to the resultant or Reading for the given area, it is *algebraically* added to the given Constant. It is evident that the algebraic sign of the Constant will always be positive or +,

Applied to an actual measurement this operation would be as follows·

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Example of Measurement.

Example 2. *Let it be required to measure the area of a plane figure drawn to a scale of say 1 in. = 40 ft. the size of which is too great to allow of measurement by the first or Exterior pole method.*

From Table 1, Plate I, we find the Setting for the scale 1 in. = 40 ft. to be 92.6, with 20.0 as the value of the Relative Vernier Unit.

Adjusting the instrument to 92.6, the given Setting, the Pole is placed at or near the center of the given figure and in such a position that the Tracer can reach every point of the periphery of the figure without moving the Pole.

In this form of measurement owing to the fact that when the area of the figure to be measured is approximately equal to the area of the Constant Circle the resulting Reading of the Planimeter is small, (being only the difference between the two quantities named) it is well to cause the Integrating Wheel, or both Integrating and Counting Wheels to read Zero, as it not only facilitates taking the final Reading but also diminishes the chance for error and admits of the algebraic sign of the Reading being more readily determined.

The Tracer having been brought to the point of beginning and the instrument either being set to a Zero Reading or the Reading taken as in the first method, the Tracer is guided and caused to trace the periphery of the figure to be measured in exactly the same manner as already described for the measurement of small areas.

At the arrival of the Tracer at the starting point after having traced the entire periphery of the given figure the Reading of the instrument is again taken. If the instrument has been brought to a Zero Reading as recommended before beginning the tracing, it is evident that the final Reading is the reading for the given figure. If the instrument had not been made to read Zero but a first Reading taken, the Reading for the given area is, as in the first method, the difference between the first and second or final Readings.

Use of the Constant.

It has already been stated that the instrument when measuring a figure with the Pole on the interior of the figure records only the Reading due to the difference in area between the area of the given figure and the area of the Constant Circle for the Setting to which the Planimeter is adjusted. It has also been explained that when the area of the given figure is *less* than the area of the Constant Circle for the given Setting the resultant rolling of the wheel will be backwards and the Reading will be a —one. If the area of the given figure be *greater* than the area of the Constant the resultant rolling of the wheel will be a forward or positive rolling and the sign of the reading will be +. The final Reading being, as has been said, the Reading due to an area which is the difference in area between the area of the figure traced and the Constant Circle must then be added algebraically to the area of the Constant Circle in order to get the total Reading for the figure traced.

If the resultant Reading is found to be a backward or — one, it shows that the area of the given figure is less than the area of the Constant Circle and adding a Constant to a — Reading is equivalent to *subtracting* the Reading from the Constant: This is to be expected since the Reading, being the Reading for the difference between the areas of figure and constant circle, when subtracted from the Constant (which is really the Reading for the Constant Circle when traced with an exterior pole) evidently gives the Reading for the given figure.

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And conversely, when the resultant Reading is +, we add a Constant to a + Reading to get the reading of the given figure, since the area of the figure is greater than the area of the Constant Circle.

In our example, let us suppose that the final Reading after tracing the given figure to be say 8,743, which we know from having kept watch of the wheel is a backward or — movement. The Reading will then evidently be 10,000 — 8,743 = 1,257, which as we have seen is —. Looking in our Table we find the value of the Constant to be 18,355. Adding these quantities we have 18,355 + (— 1257) = 18,355 — 1257 = 17,098 as the total Reading for the entire figure. Multiplying this by 20.0 the value of the Rel. Vernier Unit, we have for the area of the figure, 17,098 × 20.0 = 341,960 Sq. Ft. which is the area required.

In this example if our final Reading had been say 1,257 and we know by observation that it was a forward or positive relation we should have had

$$18{,}355 + (+ 1257) = 18{,}355 + 1257 = 19{,}612$$

and 19,612 × 20.0 would be the required area = 392,240 Sq. Ft.

The description of this method of Planimeter measurement just given will be more clearly understood at least as to the "why and wherefore" from the demonstration and discussion of the theoretical principles involved in the construction and operation of the Polar Planimeter which are given in Chapter III. The derivation and significance of the Constant are there fully explained but it has not been thought best to introduce further theoretical explanation here from fear of complicating the description just given for the practical use of the Planimeter.

The intent of the descriptions given throughout of the use of the Instrument, both of the methods of measurement already given and in the description of the use of the Planimeter in all its practical application, has been to make them so clear so as to admit of the intelligent use of the instrument in every operation without reference to the theoretical conditions involved.

Formula for Area.

In the operation just described

Let A = Area of Figure

y = Value Relative Vernier Unit.

r_1 and r_2 = First and Second Readings respectively.

C = Constant for given Scale and Setting.

Then

$$A = C + (r_2 - r_1)\, y \quad \text{---------- (1)}$$

It is evident that, as in the case of the measurement of small areas, if the actual area in Sq. Ins., of the given figure is required, it is at once obtained by multiplying by the value of the Actual instead of the Relative Vernier Unit for the given scale.

Accuracy of Results.

Of the relative accuracy in results obtained by the two methods just described there seems to be a diversity of opinion. Some authorities claim that while the second method of measurement—with Pole in the interior of the figure—may and does give results with a degree of accuracy well within the allowable error in most instances, that measurement by the first or interior pole method is much more accurate and free from probability and chance of error, and hence when required to measure

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an area whose size would preclude measurement of the whole by the exterior pole method, always divide the given large area by diagonal straight lines into constituent small areas, and measuring each independently by the first method, add, the results for the total area.

Others again claim to get equally accurate results by either form of measurement and the experience of the writer has been that there are very few instances in which with careful operating, results obtained by the second method are not of a degree of accuracy well within the permissable limit of error and entitled to confidence in their use.

It is however advisable to always use the first method when the size of the figure will permit and when the size of the given figure does not allow of the first to use the second method, checking in cases requiring great accuracy by subdividing and independently measuring the areas of the subdivisions.

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CHAPTER V.

Problems Involving Areaging.

There is perhaps no class of problem to whose solution the principle underlying the construction and operation of the Polar Planimeter is so peculiarly applicable as it is to that large class of problems so common to Engineering and experimental work, which involves the determination of the average or mean value of a variable quantity.

This class is a large and important one, and includes amongst its many forms almost every form of operation having for its object the securing of a maximum degree of accuracy in the determination of the true value of a quantity by the operation of averaging; this is best illustrated, perhaps, by the method of repetition in the case of the measurement of an angle and processes of similar nature.

To this class also belong problems such as the determination of the average or mean height of an indicator and similar diagrams, the finding of the center of gravity of a plane figure, problems involving the principles of probability and probable error, the reduction of the records of self-registering instruments, and in fact every form of problem in which the relation of two variables can be expressed in the form $A = X \times y$, and which is capable of graphical representation by an area.

Application of General Principle of Planimeter.

That principle which, as we have said, makes the Planimeter especially applicable to this class of problem, is that which we have elsewhere demonstrated and termed the "General Principle of the Polar Planimeter," and is given with its demonstration and discussion in Chap. III. To more clearly illustrate its relation in this connection, this General Principle can be stated as, *the Area of any figure measured by the Polar Planimeter is equal to the Area of a rectangle having for its base the length of the Tracer Arm and for its height a distance equal to the distance rolled by the wheel during the tracing of a given figure.*

This principle stated mathematically is what we have termed the "General Equation of the Polar Planimeter," and is repeated here on account of its especial significance in the operation under discussion.

Demonstration of Principles Involved.

The General Equation is

$$A = t \times c \times r \text{----------} (1)$$

in which

A = Area of the figure traced.
t = Length of Tracer Arm.
c = Circumference of the Integrating Wheel.
r = No. Revolutions of Wheel due to tracing the given figure.

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To illustrate the application of this Equation in this connection let us take a rectangle having an area equal to the area of any given plane figure.

The area of this rectangle will evidently be

$$A' = \text{Base} \times \text{Altitude} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Since the area of the given figure and the rectangle are equal, or $A = A'$, we have

$$t \times c \times r = \text{Base and altitude} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

If now we make the length of the Tracer Arm t equal to the base of the rectangle, we have

$$c \times r = \text{Altitude} \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

and since $c \times r$ is the distance rolled by the Wheel during the tracing of the given figure, Eq. 4 shows that the rolling of the Wheel measures the altitude of the given area whose base is equal to the length of the Tracer Arm.

If instead of a rectangle the figure is in the form of the figure given in Fig. 1 of P., Plate V, the formula for the area still holds good with the exception that the altitude in this case is the mean distance between the base A B and the curved line C D, which is evidently the mean height of the figure A B C D.

Formula 4 then becomes.

$$c \times r = \text{Mean Ht.} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

and hence to obtain the Mean Ht. of any figure of a form similar to A B C D, we have simply to make the length of the Tracer Arm equal to the length of the Base A B and trace the figure A B C D in the usual way. The number of complete revolutions and fractions of a revolution made by the Integrating Wheel during the tracing of the given figure multiplied by the circumference of the Wheel is then the quantity $c \times r$ of Eq. 5, and is the mean or average height of the given figure, since it is the distance rolled by the Wheel. The same result is obtained without making the length of t equal to the base of the figure by tracing the figure with any length base and dividing the Area thus obtained by the length of the given base obtained by direct measurement by scale, and this is the method necessarily adopted when the Planimeter used has a fixed length of Tracer Arm, as is the case in the cheaper grades of the instruments.

SPECIAL ATTACHMENT FOR MEASURING AV. HT. OF DIAGRAMS.

In the best forms of Polar Planimeter there is often a special attachment or arrangement (See Pg. 93) by which the Tracer Arm can be accurately and quickly adjusted so that its length shall be equal to the length of the base of any figure to be measured and without the necessity of measuring the base. In one form this attachment consists of two fine needle points which project upwards from the Tracer Arm, one needle point being at the Tracer and is usually the Tracing needle itself prolonged upwards, while the other is at the Carriage Pivot F and attached to the Carriage.

The adjustment of the length is then made by turning the Planimeter upside down, and having pushed the needle point at the Tracer end into the paper at one end of the base of the given figure, the Carriage is slid along the Tracer Arm until the second needle point is brought to the other

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end of the given base, when the Carriage is clamped and the instrument turned over. It is evident that by this operation the Tracer Arm length F T is made equal to the length A B of the base of the diagram.

Other forms of this attachment are modifications of the form just described and usually require a corrective factor by which the Reading for the given figure must be multiplied.

M. Coradi has adopted for his Compensating Planimeter a very simple but effective means for making this adjustment, and which is made possible by the particular form of construction of this instrument. It consists simply of making a very fine hole in the bottom of the receptacle, into which the ball joint end of the Polar Arm fits. The Tracer can then be placed at one end of the base of the given diagram and the carriage moved until the other end is seen this small hole.

Although of no more importance than many other operations made possible by the principle of the Planimeter we have been discussing, the finding of the average or mean height of an indicator diagram is perhaps that operation with which the Planimeter is most commonly associated. This use of the instrument and its value in obtaining the most accurate results has caused the production of a number of simple forms of planimeters designed especially for the measurement of indicator diagrams, and by whose use a degree of accuracy in this class of calculation is attained impossible by any other method.

Method of Measurement.

The use of the Polar Planimeter in measuring these diagrams and obtaining their mean height is exactly the same as for the plain diagram used in our description, and is as follows:

Let a b c e in Fig. 3, Plate III, be the indicator diagram from which we desire to ascertain the mean effective pressure of steam in the cylinder of the Engine from which the diagram is taken. The paper or cardboard on which the diagram is drawn should be pressed down smoothly on a very level board or table, and if, as is generally the case, the paper or cardboard is too small to contain the entire path of the Planimeter Wheel during the tracing of the diagram, a supplementary sheet of paper having a surface favorable for the purpose must be smoothly lain down down in such a position relative to the diagram that the entire path of the wheel shall be within its area during the entire tracing of the diagram.

The base of the diagram having been decided on, the length of the Tracer Arm FT is made equal to the length of the base of the diagram. This adjustment may be made in the manner already described by measuring the length of the base by scale, or, if the Planimeter has a special attachment for making this adjustment, it can be used in the manner shown in Fig. 3 of Plate III referred to.

The adjustment for length being completed, the Planimeter is placed in the most favorable position, the Tracer brought to the place of beginning already determined, and the instrument brought to a Zero Reading.

The periphery of the diagram is then traced in the direction of the hands of a watch, in exactly the same manner as for the measurement of the area of a plane figure, and the tracer having completed its circuit of the diagram, the Reading of the Planimeter is taken. Unlike the Reading for the measurement of an area however, which, as we have seen, we consider as expressed in Vernier Units, the Reading in the present case is expressed in complete revolutions and tenths, hundredths and thousandths of a revolution, the only difference between the two being in

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the position of the decimal point. This Reading then multiplied by the circumference of the wheel gives the actual distance rolled by the Wheel during the tracing of the diagram and is, as we have just proved, the average or mean height of the given diagram.

Example of Measurement.

As applied to the particular diagram in question, let us suppose that, having unscrewed the protecting caps from the attached needle points, we turn the Planimeter over, and placing the point T at the right hand end *B* of the diagram abce, we move the carriage until the needle point at F is brought to the left hand end *A* of the base *AB* and clamp the Carriage in this position.

Turning the Planimeter upright again, we place the Pole in its most favorable position, bring the Tracer to the starting point selected, adjust the instrument to Zero Reading, and trace the diagram abce in the direction indicated.

Let us suppose that at the end of the tracing we find that the Integrating Wheel has made one complete revolution and 483 one-thousandths of a revolution during the tracing of the given diagram, and that the Wheel of the Planimeter used has a circumference of 66.66 m/ms. It follows then that the required average or mean height of given diagram is :

$$1.483 \times 66.66 = 98.857 \text{ m/ms.}$$

Discussion.

When we consider that without the aid of the Planimeter this calculation for finding the mean height of these diagrams must be made by drawing a greater or less number of equidistant lines through the diagram perpendicular to the base, and measuring the intercepted length of each perpendicular separately, and from this sum obtaining the average height, some idea of the usefulness of the instrument may be arrived at.

A further proof of its accuracy and importance is gained from the fact that, on the results of the measurements of these diagrams with the Planimeter in the manner just described is based the engine and speed tests of all vessels designed for the U. S. Navy, involving in each instance the thousands of dollars in bonuses or penalties apportioned as the results of these tests.

It is, of course, evident that the use of the Planimeter in this form of operation is by no means limited to the case of Steam Engine indicator diagrams, but is equally applicable, as has already been stated, to any and every form of diagram in which the average or mean value of a variable quantity is to be obtained from a greater or less number of observed values of the variable.

Application To or Discharge of River and Similar Diagrams.

As a further example of the use of the instrument in this particular form of problem, let us take a case of very frequent occurence in engineering work—that of obtaining the average discharge of a stream or river from a set of observations extending over a given time.

In Fig. 2 of Plate IV is given a diagram taken from a Report of the United States Geological Survey, and is the graphical representation of a set of observations taken twice a day, and covering a period of one month or thirty days of the observed volumes of discharge of a river.

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In all these diagrams it makes no difference in their reduction by means of the Planimeter whether the diagram be a plotted one, or whether it be the record of a self recording mechanism, providing only that their size be such as to come within the capacity of the instrument.

In the case under discussion a line AB is taken as a base, the height of which is entirely arbitrary, provided only it does not exceed the maximum length of the tracer arm of the planimeter to be used in its reduction. This base AB is divided into thirty equal parts to correspond to the thirty days covering the period during which the observations were taken, and through each point of this division of the base perpendiculars of indefinite length are drawn.

As two observations were made each day, each one of the thirty divisions is divided into two equal parts, and through these intermediate parts other perpendiculars are drawn as shown on the given figure.

A vertical scale is now assumed in which a unit of length is made to represent graphically a certain number of thousand or million gallons discharge. In the particular diagram under description a linear inch was made to represent say one million gallons, and the rate of discharge observed each half day is graphically represented by making the length of the perpendicular which represents the day or half day on which the observation was taken equal in length to the observed rate of discharge, thus making the perpendicular above the base AB as many inches and decimals of an inch long as there was million gallons and decimals of a million gallons discharge at the time of the observation.

It is evident from this description of the construction of the diagram that it is simply plotting the "curve" of the variable to rectangular coordinates, the times of the observations being plotted as abcissae and the corresponding discharge ratio as ordinates.

If now we connect the top of each perpendicular with the top of its adjacent perpendiculars by straight lines we shall have the broken line which forms the upper side of the figure we are discussing. This brokenline is then the graphical representation of the fluctuation of the discharge of the river for the thirty days during which the observations on the discharge were made. By increasing the number of observations it is evident that the number of small lines which unite the tops of the perpendiculars will be proportionally increased while their length is diminished, and if the number of observations be very large the lengths of these connecting lines will have become so small as to be practically elements of a curve, and the broken line which forms the top of the diagram will become a curve. This is what happens when the diagram is automatically drawn by a self-recording mechanism, in which case the observations being continuous are infinite in number and the connecting lines are infinitely small.

It is evident then that both in the case of a diagram constructed as above described with a comparatively few numbers of observations, and in the case of the self-recorded diagram in which the observations are infinite in number, the average or mean discharge will be average or mean of *all* the observations and will be represented by a perpendicular whose length is the mean height of all the perpendiculars or the mean distance between the base AB and the broken line or curve EF of the given diagram.

If the Planimeter we are using is of the simple form and has a Tracer Arm of fixed length it is obvious that to obtain the length of this mean perpendicular or the mean distance between the base AB and the line EF we have simply to measure the Area ABCD of the given diagram, and since we know the length AB of the base the average or mean distance will be equal to the measured area divided by the given length of base.

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This being done we have simply to multiply this distance by the number of gallons per linear unit to which the observations have been plotted, and the result will be the average or mean discharge for the entire period through which the observations were made.

If on the other hand we have a Planimeter with an adjustable length of Arm, graduated into half millimetres and having the attachment already described, we adjust the Tracer Arm so that it is equal in length to the base AB of the given diagram and trace the diagram in the usual manner. The Reading for the given diagram multiplied by the circumference of the Wheel will then give the required mean height of the diagram in millimetres and this height divided by 25.4001, the number of millimetre is a linear inch and multiplied by one million, the vertical scale to which the observations are plotted gives the average or mean rate of discharge for the month during which the observations were made,

The application of this operation to the finding of the position of the center of gravity of any plane area is obvious, and although of very great importance will on that account not be given.

General Notes on the Method.

The two examples just given in detail illustrating the use of the Planimeter in this form of problem will serve to illustrate the method of application of the instrument to all similar forms in which the average or mean value of a variable quantity is sought, as the theory and operation is the same in all.

It will take but little consideration to discover how easily this "averaging" property of the Planimeter can be applied to any and every problem involving the principle in question and to realize how many problems there are of constant occurrence in the engineer's practice to whose solution this property of the Planimeter makes the instrument particularly adapted. Some of these we have already mentioned, and the list could be extended almost indefinitely. Enough, however, has been already said to show that even were the application of the Planimeter limited to the particular form of operation just described, its almost incredible accuracy of operation and the enormous saving in time and mental effort effected by its use would alone make it an indispensable aid in the engineer's labors.

In the highest form of the Planimeter, of which the Integraph and Integrator are examples, the principles just described have been developed even beyond the capacity of the Polar type we are dealing with, so that in addition to an ability to perform the operations possible to the simpler form they are capable of solving problems involving the finding of moments of inertia and stability, determination of stresses in framed structures, centres of gravity and gyration, and many others too numerous to mention.

This class of Planimeter will be more fully described later when the higher forms of the instrument are taken up for discussion.

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CHAPTER VI.

Quantities of Materials.

Of the many and varied problems with which the Engineer of whatever branch is concerned, there are few if any in which the calculation of quantities and volumes of materials does not enter to a greater or less extent. While this is true of all branches of Engineering work it is more especially true with the particular class of problem coming within the province of the Civil Engineer, and it is easily true that in the great majority of cases with which he has to deal by far the greater portion of the time and labor involved in the solution of any given problem is occupied in the making of these calculations.

The importance of these calculations and the necessity of a high degree of accuracy in their results is too evident to need expression and but illustrates the need and enormous value of any aid capable of lessening the time and mental labor necessarily involved. The value of such aid is even more evident when we stop to consider how often problems arise in which the final decision in any given case is dependent on the results of the calculation of comparative costs: such for example as the final location of a section of railroad when several locations are feasible—the choice of a Reservoir site, and innumerable problems of similar nature. In too many instances the enormous amount of labor and time necessary for the calculation of the relative quantities of materials involved in making such comparative estimates limits the number considered, and in many instances causes the selection of a location which is afterwards found to have been by no means either the best or the cheapest.

A further result of the use of any efficient aid in this form of calculation is easily seen to be a much more accurate and intelligent preliminary report and estimate in any given case and the lessening of the drudgery involved in the making of the inevitable "monthly estimate."

The attempt to supply such an aid as we have been discussing is seen in the almost innumerable number of Tables, diagrams and similar devices which have appeared, each claiming to best serve the purpose intended, and many of which are of more or less assistance to the Engineers: but of the many with which the author is familiar there are none which can from any point of view compare with the Polar Planimeter. In degree of accuracy, saving of time and labor, and adaptability to every condition, the value of the Planimeter in this connection cannot be over estimated.

While perhaps the most frequent use of the Polar Planimeter in this class of problem is in computing the volumes of Earth work or other material from cross-sections, since these computations are the most frequent and occupy the larger share of the Engineer's time, the instrument is by no means limited to that particular form of operation, but is adapted with equal facility and accuracy to *every* calculation in which the measurement of volumes or quantities of materials is involved. On this account and on account of the invaluable aid of which the Planimeter is capable in *every* application of this form of computation the use of the

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Instrument will be described in those calculations of most frequent occurrence in the Engineer's practice.

Very complete factors for the immediate adjustment of *any* Planimeter and for *all* scales are given in the Tables for all of these operations and applications and these factors together with the following descriptions of the use of the Planimeter will at once admit of the intelligent use of the instrument in all calculations of similar nature.

The operations selected to illustrate the use of the Polar Planimeter in the various calculations of the quantity or volume of materials are as follows and will be taken up in the order here given :

I. VOLS. FROM CROSS SECTIONS.
   1. General Methods of Measurement.
   2. Vols. of Single Prismoids.
   3. Vols. of Continuous Prismoids.
      a From Plotted Sections.
      b From Field Notes Direct.

II. VOLS. FROM ORIGINAL CONTOURS.
   1. General Methods of Measurement.
   2. Grading over Extended Areas.
   3. Contents and Areas of Reservoirs.
   4. Vols. from Displacement Diagrams.

III. VOLS. FROM ORIGINAL AND FINAL CONTOURS.
   1. General Methods of Measurement.
   2. Grading over Extended areas.
   3. Vols. of Material in Reservoir and Similar Construction.

IV. SPECIAL METHODS OF MEASUREMENT.
   1. General Methods of Measurement.
   2. Vols. of Brickwork in Sewers, Walls, Tunnels, etc.
   3. Wt. of Iron and other Metals.

1. METHODS OF MEASUREMENT IN GENERAL.

MEAN END AREA AND PRISMOIDAL METHODS OF MEASUREMENT.

The two methods in general use for obtaining the volume or quantity of any materials in Engineering or Construction work are what are known as the "Average" or "Mean End Area" method and the Prismoidal method.

Of these, the first or Method of Mean End Areas is only accurate in the very few instances, when conditions may make it a special form of the second or Prismoidal method. While often used, and indeed, having its use in some States legalized by statute, the method is at best but an approximation, and when used except as an approximation, is liable to work great injustice to either the Contractor or to those whose interests are represented by the Engineer. In almost every case the results of the measurements of a quantity by this method are larger than the true quantity, and although attempts to counteract this are made by applying a correction, the error not being a constant one can not be eliminated, and is often productive of any amount of subsequent trouble and disagreements.

The extended use of this Average End Area Method is due entirely to the rapidity with which calculations for the determination of quantities of materials can be effected, as compared to the second or Prismoidal

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Method, which while rigid and mathematically accurate, necessitates a much longer time to obtain any desired result than does the first.

Owing to the extended use of the Average method just mentioned, and to the fact that the method has a certain value where confined to rough preliminary estimates or approximations, the use of the Planimeter in connection with the solution of problems by that method will be included in our discussion, although it will later be seen that the use of the Polar Planimeter in all this form of computation entirely eliminates every advantage as to time possessed by the Average Method, allowing an absolutely accurate result by the Prismoidal method to be obtained in exactly as short a time as an inaccurate result by the Average Method.

2. VOLUMES OF SINGLE PRISMOIDS.

While the following descriptions and demonstrations are entirely general and apply to every form of prismoidal calculation, practical examples have been used to illustrate the use of the Planimeter in its various applications, and the examples selected for illustration made as typical as possible, so that no difficulty will be experienced in applying the methods to particular cases.

Application of the Prismoidal Formula.

As generally written the Prismoidal formula is:

$$V = \frac{L}{6}(A_0 + 4\,A_m + A_1) \text{--------} (1)$$

in which

V = Vol. of Prismoid in Cu. Ft.
L = Dist. in Ft. bet. End Sections.
A_0 = Area in Sq. Ft. of one End Area or Section.
A_1 = Area in Sq. Ft. of other end Area or Section.
A_m = Area in Sq. Ft. of a Section of the Prismoid midway bet. the two end sections, the lengths of the sides of this middle Section being the means of the lengths of the corresponding sides of the two End Sections.

When the volume of the given Prismoid is required in Cu. Yds. Eq. 1 becomes:

$$V \text{ Cu. Yds.} = \frac{L}{6 \times 27}(A_0 + 4\,A_m + A_1) \text{--------} (2)$$

If L be taken as 100 ft., which it usually is in the calculation of railroad excavation, Eq. 2 becomes:

$$V \text{ Cu. Yds.} = \frac{100}{6 \times 27}(A_0 + 4\,A_m + A_1) \text{--------} (3)$$

In Fig. 1 of Plate IV, let us suppose that we have plotted the two End Sections A_0 and A_1 as there shown; Section A_0 being ABCPD and Section A_1 being ABHNG, both sections being plotted to a scale of say 1 inch = 8 ft, and both sections having, as in all railroad sections, the same width of road bed and the same ratio of side slopes.

Since these sections are plotted to a lineal scale of 1 in. = 8 ft., 1 sq. in. of section will represent $8^2 = 64$ sq. ft., and we have as the Volume represented by the two sections:

$$V \text{ Cu. Yds.} = \frac{100 \times 64}{6 \times 27}(A_0 + 4\,A_m + A_1) \text{--------} (4)$$

or by reduction V Cu. Yds. $= 39.5061\,(A_0 + 4\,A + A_1)$ --------(5)

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Method of Plotting the Cross Sections.

It is evident from the definition given above that, since the area AB CPD is the A_0, and the area ABGHN is the A_1 of our Equations, if we bisect the distances CH, PN, DG on our diagram and draw the lines FM and ME, the resulting area ABFME will be the A_m of the Equations, since by so doing we have drawn an area the lengths of whose sides are the means of the lengths of the corresponding sides of the two end sections A_0 and A_1.

If, now, we suppose the sum of the three areas A_0, 4 A_m and A_1 to be say 1 Sq. Inch, and subtitute in Eq. 5 we have:

$$V \text{ Cu. Yds.} = 39.5061 \times (1) \text{--------} (6)$$

or

$$V = 39.5061 \text{ Cu. Yds} \text{--------} (7)$$

Eq. 7 then shows that when the sections of any given prismoid are plotted to a scale of 1 in. = 8 ft., each sq. in. of actual area represented by the term $(A_0 + 4\,A + A_1)$ will represent a volume of 39.5061 Cu. Yds., the prismoid being 100 ft. in length.

If, then, we adjust the Planimeter to that Setting which will cause the Instrument to record 3950.6 Vernier Units when tracing an actual area of 1 Sq. inch, and with the instrument so adjusted we trace continuously and successively the plotted sections as indicated in the expression $(A_0 + 4\,A_m + A_1)$—that is, tracing the area ABCPD once, the area ABFME four times and the area ABHNG once—it is evident that the number of Vernier Units recorded for that tracing when multiplied by .01, the value of the Relative Vernier Unit, will be the number of Cu. Yds. contained in the given prismoid.

Method of Measurement.

From the above explanation it is evident that to measure the volume in Cu. Yds. of *any* prismoid with the Planimeter, we plot the two end areas to any desired scale and from them plot the intermediate Section as above described, and as shown in Fig. 1 of Plate IV. Then having adjusted the Planimeter to the Setting given in the Tables for the scale to which we have plotted the Sections, we trace continuously the two end Sections each once and the middle or Mean Section four times, and having taken the total Reading due to the tracing, we multiply it by the Vernier Unit given in the Table for the scale in question. The product of the Reading thus obtained when multiplied by the Relative Vernier Unit will be the desired volume of the Prismoid.

It is evident that in the case of a prismoid having a less length than 100 ft., the operation would be exactly the same except that having obtained the volume of the prismoid in exactly the same manner as though it were 100 ft. long, we simply multiply the volume thus obtained by the actual length of the given prismoid expressed as a per cent. of 100 ft.

Owing to the strict importance of a thoroughly intelligent understanding of the operation just described, it will be best to illustrate it more fully by the following example:

Let us suppose that we desire to measure the volume in Cu. Yds. of a Section of Railroad excavation, the length of the Section being say 73 ft., and that we have the field notes of the Cross-sections at both ends.

Having selected the Scale of 1 inch = 10 ft. as our plotting scale, we proceed to plot both end sections one upon the other, as shown in Fig. 1 of Plate IV, ABCPD being one End Section and ABHNG the other End Section plotted to the scale selected.

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We then find by scale the middle points of the lines CH, PN and DG and connect these middle points by the lines FM and ME, as shown.

Looking in the table headed Volumes by Single Prismoids (Table 7), we find that the Setting for the scale of 1 in. = 10 ft. is 307.0. Adjusting the Planimeter to this Setting, and bringing the instrument to a Zero Reading, we place the Pole in the most favorable position relative to the plotted Sections and bring the Tracer to A as the place of beginning for the tracing. Then carefully, and with regard to the rules applying to all cases of Planimeter measurements, we proceed to trace the Section ADPCB. Instead of stopping when we have arrived again at the beginning point, the tracer is carried on past A and the Section AEMFB is carefully traced four times, after which the tracer is made to trace the Section AGNHB once. The entire tracing is performed without stopping and without regard to or the taking of any intermediate Readings, the only watching of the recording wheels being to see if the Recording Wheel has passed the O mark and begun a new revolution during the progress of the tracing.

The tracing of the three Sections having been accomplished in the manner described, the Reading of the Planimeter for the entire tracing is then taken. Looking again in the proper table (Table 7), we find the value of the Relative Vernier Unit for the given scale to be .9.

Let us suppose the reading already taken for the entire tracing to be say 12472.0 Units : the volume of the given tracing, were its length /100 ft., would then be $12472 \times .9 = 11224.8$ Cu. Yds., but since the length of the measured prismoid is only 73 ft., the required volume is evidently only 73 per cent. of what it would be were it 100 ft. long, and hence the volume required is 73 per cent. of 11224.8 or $11224.8 \times .73 = 8198.1$ Cu. Yds.

Accuracy of Results.

If the Planimeter used, instead of having an adjustable arm, should be the simpler form of instrument and only capable of giving areas in one of unit, say sq. inches, it is evident that a considerable modification would be necessary from the operation just described. In fact, the operation would have to be exactly the same as it would have to be were no Planimeter at all used except for the saving in time and effort effected by the Planimeter in measuring the number of sq. inches contained in the three sections. As to the degree of accuracy with which these results are obtained in the manner just described, it is evident that much depends upon the accuracy of the plotting of the Sections, as well as upon the care with which the tracing of the plotted sections is accomplished. Repeated trials made to test the accuracy of results obtained in the manner described have shown that, with ordinary care in the plotting of the Sections, and with that attention to the requisites for correct tracings which should be taken in all instrumental work, the error in the measurement of a prismoid need never be greater than two cu. yds. in one thousand or 0.2 of 1 per cent.

In fact, even in cases where time is a very necessary factor, and where the rapidity with which the measurements are made precludes the attaining of any but a very ordinary care in operating, the results obtained have always proved to be far within the accuracy of any but the most accurate and precise field work and quite equal even to that.

As to the relative saving of time and labor of the Planimeter measurements and the same measurements made by any other known method, either table or diagram, that the proportion in favor of the Planimeter is easily within the ratio of 1 to 3 is easily demonstrated.

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As this subject of accuracy must come up for more detailed discussion later, it will not be enlarged upon here further than to say that when once the Engineer has used the Planimeter as an aid in this form of computation he will never again return to any—even the most accurate—which he may have used before, a remark which applies with equal truth to every form of calculation coming within the range of the Polar Planimeter's capacities.

3. Volumes of Continuous Prismoids.

A. Volumes from Plotted Sections.

The method just described for finding the volume of any prismoid by means of the Polar Planimeter while applicable to the case of a single prismoid, or where the prismoids to be measured are not continuous, can be modified in such a way when the volume of several continuous prismoids is required as to effect an even greater relative saving in time and labor than in the case of the single measurement.

The first record of the use of the Polar Planimeter for the measurement of a number of continuous prismoids by one operation with which the writer is familiar, appeared in an article describing the method by Clemens Hershell, C. E., in the Journal of the Franklyn Institute for April, 1874, and is essentially the same as that about to be described although worked out entirely independently and without knowledge of Mr. Hershell's article.

Deduction of Formula.

To develop a form of the Prismoidal formula which will cover the case of obtaining by one operation the volume of a number of continuous prismoids, let us suppose that we desire to find the total volume of earth excavation in a railroad section several hundred feet in length, cross-sections of which have been taken as usual at each one hundred feet.

Let us suppose there are n of these prismoids each of which is 100 long, and that beginning with the first end section we denote the successive cross-sections by the letters A_0, A_1, A_2, A_3----A_{n-1} and A_n, the subscript of each A denoting the number of prismoids up to that section and n being an even number.

It is evident that with $L = 100$ and treating each A having an *even* subscript as the end section of a prismoid, $2 \times L = 2 \times 100$ or 200 ft. long, the intermediate As having an odd subscript would correspond to the A_m. of Eq. 1, Pg.

Taking then the prismoid whose end sections by this arrangement are A_0 and A_2 and making A_1, the middle section, we have for the volume in cubic yards of the first prismoid

$$V_1 = \frac{2 \times 100}{6 \times 27} (A_0 + 4\,A_1 + A_2) \text{--------} (8)$$

Taking the next prismoid whose end sections are evidently A_2 and A_4, and whose middle section is A_3. and denoting its volume in cubic yards by V_2 we have

$$V_2 = \frac{2 \times 100}{6 \times 27} (A_2 + 4\,A_3 + A_4) \text{--------} (9)$$

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Similarly for the third prismoid for whose volume we should have

$$V_3 = \frac{2 \times 100}{6 \times 27}(A_4 + 4\,A_5 + A_6) \text{.....(10)}$$

and for the last prismoid

$$V_n = \frac{2 \times 100}{6 \times 27}(A_{n-2} + 4\,A_{n-1} + A_n) \text{.....(11)}$$

Since the factor $\frac{2 \times 100}{6 \times 27}$ is common to all we should have the total volume of the n prismoids.

$$V_1 + V_2 + V_3 + V_4 + \text{....} + V_n = \frac{2 \times 100}{6 \times 27}(A_0 + 4\,A_1 + 2\,A_2 + 4\,A_3 + 2\,A_4 + \text{.....} + 4\,A_{n-1} + 2\,A_n) \text{.....(12)}$$

or by reduction and arranging

$$\text{Total Vol.} = \text{V Cu. Yds.} = \frac{2 \times 100}{3 \times 27}\left(A_0 + 2\,A_1 + A_2 + 2\,A_3 + A_4 + \text{....} + A_n - \frac{A_0 + A_n}{2}\right) \text{.....(13)}$$

or

$$\text{Total Vol.} = \text{V Cu. Yds.} = 2.4691\,(A_0 + 2\,A_1 + A_2 + 2\,A_3 + A_4 + \text{....} + A_n - \frac{A_0 + A_n}{2}) \text{.....(14)}$$

which is the formula desired. It is at once seen that this general formula can be written as

Total Vol. = V Cu. Yds. = 2.4691 (Sum Even A's + Twice Sum Odd A's — ½ Sum Extreme A's).....(15)

To employ this formula let us suppose that the railroad section whose volume is desired is say 600 feet long. Having selected a scale, say 1 inch to 8 feet, we proceed to plot the consecutive cross-sections, superimposing them one on another as shown in Fig. 3 of Plate IV, and denoting each cross-section by the letter A with the proper subscript showing its number from the first end section.

Since we have plotted our cross-sections to a linear scale of 1 in. = 8 ft. each square inch of actual area of plotted cross-section will represent 8^2 or 64 square feet, and introducing this factor in the general Eq. 14 we have the particular formula for this scale

$$V = 2.4691 \times 64 \times (A_0 + 2\,A_1 + 2\,A_2 + 2\,A_3 + A_4 + 2\,A_5 + A_6 - \frac{A_0 + A_6}{2}) \text{.....(16)}$$

or

$$V = 158.02\left(A_0 + 2\,A_1 + A_2 + 2\,A_3 + A_4 + 2A_5 + A_6 - \frac{A_0 + A_6}{2}\right) \text{..(17)}$$

In Eq. 17 it is seen that if the entire quantity inside the parenthesis should reduce to an actual area of 1 Sq. Inch the resulting equation would be

$$V = 158.02 \times (1) = 158.02 \text{ Cu. Yds} \text{.....(18)}$$

which shows that *if we have plotted all our cross-sections to the scale*

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of 1 inch = 8 feet, that every square inch of actual area of cross-section so plotted which is represented by the quantity within the parenthesis, represents a Vol. of 158.02 Cu. Yds.

It is evident then that if we adjust the Planimeter to that Setting which will give a Reading of 1580.2 Vernier Units when tracing an actual area of 1 square inch, and then proceed to trace the various plotted cross-sections continuously and in the manner indicated by the quantities inside the parenthesis of Eq. 17, the resulting Reading of the Planimeter at the completion of the entire tracing when multiplied by the value of the Relative Vernier Unit, which in this case is 0.1, will be the total volume in cubic yards of the 6 prismoids or the entire total volume of the section between the end sections A_0 and A_6.

Method of Measurement.

The actual method of tracing as indicated by the quantity within the parenthesis would be as follows: After having adjusted the Planimeter as described to that Setting which will give a Reading of 1580.2 units for 1 square inch of actual area, we bring the instrument to a Zero Reading, and having placed it in its most favorable position as elsewhere described, we select a beginning point and trace the plotted sections marked A_0 and A_6, as shown on Fig. 3 of Plate IV as already mentioned. After having traced A_0 and A_6 once each as described, the reading of the Planimeter due to the tracing of these two sections is then taken and being divided by 2 is set down for later use.

The Planimeter is again set to a Zero Reading, and following the operations indicated by the quantities within the parenthesis of Eq. 16 we trace A_0 once, then A_1 twice, then A_2 once, A_3 twice, and so on, tracing each A having an *odd subscript twice* and each A having an *even subscript once* until we have arrived at and traced the last section A_6 once as indicated.

The entire operation of tracing these sections in the manner indicated is continuous, no intermediate Readings other than the one for the quantity $\frac{A_0 + A_n}{2}$ as already described are taken, and no attention need be paid to the Wheel movements throughout the entire operation except to note whether and how many times the counting wheel may have passed its 0 mark and begun a new revolution.

At the completion of the entire tracing the Reading of the Planimeter is taken, and from this Reading is subtracted the Reading first set down for the quantity $\frac{A_0 + A_5}{2}$ This subtraction having been made the resulting quantity is multiplied by the Relative Vernier Unit given for the scale to which the cross-sections are plotted.

The product is then the the entire volume of the six prismoids or the volume of the R'y. section between the Sections A_0 and A_6.

Example of Measurement.

To make the above description clearer, let us suppose that in the case of the six prismoids of railroad excavation we have been discussion, we have plotted the cross-section to the scale selected, 1 inch = 8 ft., and that Fig. 3 of Plate IV. are the plotted end areas or cross-sections.

Looking in the table headed "VOLS. OF CONTINUOUS PRISMOIDS," we find that the Setting there given for the scale of 1 inch = 8 ft. is 268.8.

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Adjusting the Planimeter to this Setting we place the instrument in its most favorable position and bring all the wheels to a zero Reading.

Selecting the point A as a beginning point we then trace the extreme end sections, A_0 and A_6, each once—the tracing being continuous. At the completion of this tracing the Reading of the wheels is taken which we will suppose in this case to be say 2340 vernier units. Dividing this Reading by 2 gives $\frac{2340}{2} = 1170$ which is the value of the expression $\frac{A_0 + A_n}{2}$ of Eq. 14, or in this particular case of the expression $\frac{A_0 + A_6}{2}$ of Eq. 16.

We then reset the Planimeter to a zero Reading and perform the other operations indicated by the other terms of the quantities within the parenthesis : that is, we carefully and in the manner prescribed for the tracing of all plane areas with the Planimeter trace continuously and in one operation all of the plotted A's which have an even number as a subscript which in this particular case are A_0, A_2, A_4 and A_6. Then without stopping or taking any intermediate Reading we proceed to trace twice each plotted A which has an odd number for a subscript which in this case are A_1, A_3 and A_5.

When the tracing as described is completed we take the Reading of the Planimeter which we will say is 24,950 Vernier Units. From the 24,950 Vernier Units which evidently represent the value of the terms $A_0 + 2A_1 + A_2 + 2A_3 + A_4 + 2A_5 + A_6$, we subtract the 1,170 already found which represents the value of the term $\frac{A_0 + A_6}{2}$. Hence $24,950 - 1,170 = 23,780$ is the total number of Vernier Units due to tracing the cross-sections in the manner indicated.

Looking in Table No. 8 we find the value of the Relative Vernier Unit to be 2.0 for the scale 1 inch = 8 ft.

Multiplying the total number of Vernier Units by the given value of one Vernier Unit we have $23,780 \times 2.0 = 47,560$ Cu. Yds., which is the total volume of the six prismoids measured.

In Formula 14 and 15 deduced for the measurement of the volume of any number of continuous prismoids, it will be remembered that the value of L or the length of a single constituent prismoid was taken as 100 ft., and since two such single prismoids were taken as forming a larger prismoid the length of the larger prismoid thus formed was of course 2 L or 200 ft.

In calculating the factors given in Table No. 7 for the measurement of continuous Prismoids the distance apart of the end Sections has been taken throughout as being 100 ft. as in the example cited.

In rock and similar excavation the cross-sections which form the A's of the Equations Nos. 14 and 15 are frequently taken much closer together —the distance between the cross-sections often being 50 or 25 ft. or even less.

In such cases the measurements are made in exactly the same manner as when they are 100 ft. apart the factors given in the table being used exactly as in the case of the 100 ft. sections and the correction made for the shorter length between cross-sections by simply multiplying the result obtained by using the Table factors by the length between the end sections for the particular case in question expressed as a per cent. of 100 ft.

Thus if the cross-section interval in any case should be say 30 ft. instead 100 of ft., the final result obtained by using the factors in the Table

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would be multiplied by $\frac{30}{100} = .3$ to obtain the desired volume of prismoids.

It is evident from the above description of this form of measurement that there are no restrictions as to the number of prismoids except when the quantities become too large to keep track of conveniently and with the provision that the number measured by one operation shall be an *even* number,

Another limiting condition would also evidently be in the more or less confusion and consequent greater liability to error due to plotting too many sections over one another as described. This however can be partially remedied by using different colored inks for the plotting of the various sections.

Accuracy of Results.

As to the degree of accuracy in results of measurements made in the manner just described, it is evident that the nearer together the cross-sections are taken and the nearer will the prismoids conform to the actual condition of tbe surface of the ground, but in *every* case the accuracy of the measurement with the Planimeter is far within the accuracy of the cross-sections and of the field work and with accurate data and accurate plotting the volume of excavation found by this method will be well within the degree of accuracy obtainable by any other known means.

As to the saving in time and labor, it is too evident to need any discussion and a statement of the results of comparative tests of this with other methods would seem like exaggeration until the reader had tested the matter for himself.

Mr. Herschell in the article already referred to says as the results of comparative tests made by himself that the "saving of time derived from the use of the Planimeter over that required by calculating from tables or from formulae, was as 1/2 to 4/5", and that the degree of accuracy practically attained "surpasses what in any odinary work has hitherto been considered practically attainable." He also estimates the probable error as not greater than two yards in one thousand. It will take but little experience with this form of calculation to prove all of these estimates both as to saving of time and accuracy to be well upon the conservative side.

3. Volumes of Continuous Prismoids, Cont'd.

b. Volumes from Field Notes direct.

In the method just described for measuring the volume of any number of Continuous Prismoids by one operation, the cross-sections as will be remembered were plotted to some desired scale, each cross-section for convenience in plotting and tracing being superimposed on the others in the manner shown in Fig. 3 of Plate IV.

As was also stated in the description of the method, when the number of prismoids included in one operation became very large there was apt to be some confusion owing to the large number of lines in one diagram, thus bringing in a liability of error due to the possibility of tracing the wrong outline.

Advantages of the Method.

The method about to be described has not only all the advantages possessed by the first method but the further advantage of a very large saving in time and labor due to the fact that the method does not require

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that any of the cross-sections shall be plotted: it is also evident, that owing to this fact the possibility of error due to the mulitiplying of lines in the superimposed plotted cross-sections is entirely eliminated and that any number of prismoids can be measured by one operation with exactly the same degree of accuracy as can a single prismoid. The value of these advantages is self-evident and makes this method by far the most accurate and rapid of execution of any yet devised, while being at the same time so simple and so easily executed as to require but little experience to employ it with the greatest confidence and facility.

The method is exactly the same as the first both mathematically and in execution except that unlike the first method the cross-sections are not plotted. All factors for both methods are the same so that the table headed VOLUMES OF CONTINUOUS PRISMOIDS being Table No. 8 of Plate V is used for both.

Preparing the Templet.

Having decided upon one of the scales commonly used for plotting Cross-sections in railroad and similar work, a sheet of Cross-section paper ruled to the scale chosen is carefully pinned down to the drawing board as shown in the diagram on Plate VI which illustrated this operation. A heavy vertical line near the center of the sheet of Cross-section paper is then selected as a center line and a corresponding heavy horizontal line near the bottom of the sheet is selected as the base of the Cross-sections.

The width of road-bed and ratio of side slopes of the Cross-sections being of course, known, a template is prepared for that particular width of bed and slope ratio as shown on the diagram mentioned.

This template may be of good stiff card or bristol board but what is by far more preferable and accurate is one made of transparent celluloid or "Xylonite" which is manufactured and supplied only by the Keuffel & Esser Co. of New York City and from whom templates of this description for any desired scale and for any width of road-bed or ratio of slope desired can be obtained.

In the diagram mentioned (Plate IV) the scale of the Cross-section paper is 1 inch to 8 feet, while the template shown is cut to that scale and is for a road-bed having a width of thirty-two feet and a slope ratio of $1\frac{1}{2}$ to 1.

A line cut in the template at the center point of and at right angles to the base permits placing the template on the Cross-section sheet, so that the road-bed of the template and its center line will coincide with the horizontal and vertical lines of the Cross-section selected for the base and Center line of the sections.

The template having been thus adjusted to the cross-section sheet it is securely fastened in position by thumb tacks through the template and paper as shown on the diagram.

Having adjusted the Planimeter to the Setting in Table 8 for the scale 1 in. = 8 ft. the Planimeter is placed in the proper position on the Cross-section sheet, the Tracer brought to the point of beginning and the instrument set to a Zero Reading.

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Form of Field Notes.

In the particular case we are using for illustration let us suppose the cross-section notes in our field book to be as follows:

Station 0—	34.0 / 12.0	c / 16.0	37.0 / 14.0
Station 1—	35.2 / 12.8	c / 17.0	38.5 / 15.0
Station 2—	37.3 / 14.2	c / 17.8	39.7 / 15.8
Station 3—	39.1 / 15.4	c / 18.8	41.2 / 16.8
Station 4—	40.9 / 16.6	c / 20.2	43.6 / 18.4

which is the form of all cross-section notes and in which the numbers *above* the horizontal lines are the number of feet out on each side of the Center line that the slopes of the sections intersect the surface of the ground, while the figures under the lines give the height of those points above the finished grade of the road-bed. For example, at Station 0 the surface of the ground at the Center line at this section is 16.0 feet above the finished grade while the side slopes, which have a slope ratio of 1½ to 1, intersect the surface of the ground, the right hand slope at 37.0 and the left hand slope at 34.0 feet respectively from the Center line, while the elevation of these points of intersection are 14.0 ft. and 12.0 respectively above the finished grade of road-bed.

Method of Measurement and Use of the Template.

It is evident that when the template is adjusted to the Cross section sheet in the manner described, the tracing of each cross-section with the Planimeter is easily accomplished from the mere inspection of the field notes of each section: the width of road-bed and slope ratios of all the sections in any given case being the same it is a very easy matter with the template to locate on the cross-section sheet both the points of intersection of each side slope with the surface of the ground and the center height and to conduct the tracing needle from one to another of the points thus located, the edges of the template causing the needle to accurately trace the other three sides of the section. For example, suppose we wish to trace the Cross-section at Station 0 in the manner described, the operation would be as follows: Having selected a point of beginning say the point of intersection of road-bed and the left hand slope, we adjust the planimeter to its most favorable position and place the tracer at the point of beginning. We then begin the tracing by moving the tracer along the edge of the template up the left hand slope of the section until we have arrived at the point on the cross-section sheet indicated by the expression. $\frac{34.0}{12.0}$

The mental location of this point, which is the point of intersection of the given slope with the surface of the ground, is made with the greatest ease and rapidity by simply determining with the eye the horizontal

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line at the numbered center line which represents a height of 12 ft. above the Road bed and following it until it intersects the edge of the template at the side slope.

The needle being brought to this point, it is then conducted along an undrawn straight line to that point on the center line which is 16.0 above the road bed. From this it is conducted in a straight line to that point on the *right hand* slope, where the horizontal line 14 ft. above the road bed intersects the right hand slope, which point is indicated by the expression $\frac{37.0}{14.0}$; and from this point the tracer is simply carried along the edge of the template to the point of beginning, thus tracing the outline of the given cross section.

It will take but little experience to enable the three surface points of each section to be accurately located mentally and with a rapidity which should cause no stop or hesitation in the tracing and the measurement of volumes and quantities by this method, and with the use of the template will be found to be quite as accurate as is attainable with the plotted cross sections and without the time and mental effort expended in plotting the Cross-sections necessitated by the first method.

As already stated, the operations involved in both plotted section and template measurements are the same and have already been explained in detail; they consist in adjusting the Planimeter to the Setting given in the tables for the scale of the Cross-section paper used and then tracing once each the first and last cross-sections of the set of prismoids which are to be included in the one measurement, dividing the Reading due to the tracing of these two end sections by two and setting the resulting value down as a partial result. Then, as previously explained, having readjusted the Planimeter to a Zero Reading each *even* numbered section is traced once and each *odd* numbered section traced twice, the entire operation being continuous and without intermediate Reading; and after subtracting from the total Reading the value first set aside, the result is multiplied by the tabular value of the relative vernier unit for the scale used, and the product thus obtained is the total volume of the prismoids measured.

In rock work, or where a greater degree of accuracy is desired in obtaining the volumes of materials, it is often customary to take the field notes of the Cross-sections in greater detail by including in the cross-section notes the elevation of the surface of the ground above the Road bed at a distance out on each side of the Center line equal to one-half the width of the road bed, thus making the cross-sections "five level sections," instead of "three level sections," as used in the above example.

The location of these two points is quite as accurately and easily found on the cross-section sheet as are the other three, and admit of "five level sections" being traced quite as readily as are the "three level."

It may be urged, and especially in the case of large sections, that in tracing the surface of the ground of each section, since no lines are drawn, the tracer may deviate more or less from a straight path between the slope stake and the Center or the Center and the other slope stake, but it is readily seen that the actual surface of the ground is rarely a straight line, and that the deviating path of the tracer would probably conform to the actual surface with quite as much accuracy as though following a drawn straight line connecting the points.

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Accuracy of Results.

What has been said as to the attainable degree of accuracy in results by the method of plotted cross-sections applies equally to the method just described when the template replaces the plotted sections and the measurements are made from the field notes direct.

To test the accuracy of this form of measurements, the volume of a section of Railroad excavation six hundred feet long, of which the notes of five cross-sections are given on page 71, was calculated very accurately by means of the prismoidal formula. The volumes of the six prismoids were then measured continuously by the Polar Planimeter in the manner just described, the scale of the cross section paper and template used being 1 inch to 8 feet. In the measurement of these prismoids with the Planimeter no attempt was made to secure a degree of accuracy higher than attainable by average care in the tracing and use of the Planimeter in this class of work, the design being to have all the conditions of the measurement the same as would be the case in any ordinary measurement or use of the instrument.

The results of the two measurements of the 600 ft. section were as follows :

Volume by Prismoidal Tables..... 23,326 Cu. Yds.
Volume by Planimeter ..... 23,294 Cu. Yds.

making the difference in volume as given by the two methods of measurement

$$23{,}326 - 23{,}294 = 32 \text{ Cu. Yds.}$$

a difference so small as to be far within the strictest allowable limit for this class of work and showing a degree of accuracy in the Planimeter method far higher proportionately than the field operations by which the notes were obtained.

The saving in time and labor as between the two methods was also shown by this test to be well within the proportion of 1/2 to 4/5 already mentioned in the case of the method by plotted end sections ; in fact, that proportion was almost doubled and with the further very important consideration that by this method the probability of error is reduced to a minimum.

The value of the Polar Planimeter in the class of measurements we have been discussing is so great that when once used and its capacity for furnishing results unequalled in accuracy and saving in time and labor by any other known method once recognized, the Engineer will never again return to the old methods ; a fact requiring but a very short experience with the Planimeter to amply demonstrate, not alone of the instrument in this form of operation, but in *every* form of operation to which it is applicable.

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CHAPTER VII.

Quantities of Materials (Cont'd.).

II.—VOLUMES FROM ORIGINAL CONTOURS.

1—General Method of Measurement.

Although perfectly general and applicable to all forms of prismoid, we have so far limited our discussion to that particular form of prismoid occurring most frequently in the work of the Railroad or Canal Engineer, since it is in this form of prismoid that by far the greater portion of the quantities to be measured are presented.

In the examples thus far given to illustrate the use of the Planimeter in the measurement of the volumes of materials, the prismoids into which the material to be measured have been divided have had vertical end sections and their length as the greatest dimension.

There is, however, a very large class of problems of costant occurrence in the Engineer's practice in which the material to be measured, instead of being divided in the form of prismoids just described, is considered as divided into a number of continuous prismoids having more or less extended horizontal areas as bases and a very short vertical height between these bases or end areas as the length. These prismoids in most instances are formed by the passing of several parallel, equidistant, horizontal planes through the material to be measured, the sections made by these planes being the base of the prismoids, and the vertical distance between them the lengths.

From this it is readily seen that the form of measurement about to be described by which the volume of the prismoids thus formed is obtained, will include all problems involving the measurement of quantities of materials in operations, such as grading over extended areas, the measurement of borrow pits, volumes of materials in dredging operations, displacement diagrams, contents of reservoirs and many other problems of similar nature.

While the principle involved in the measurement of quantities is the same for all problems included in the class we are discussing, the general method of measurement can often be modified in the case of some particular application or application to some particular problem so as to simplify it for that operation and lessen the time and labor involved in the measurement.

To illustrate this and to show the use of the Planimeter in the various special applications referred to, several of the more important and most frequently occurring problems of this class will be taken up and the use of the Polar Planimeter in each will be explained.

Very complete Tables giving the Settings, Constants and other factors for the immediate adjustment of the Planimeter for all of the various forms of measurement described, and for all scales are given among the Tables. In those cases where the same Table is used for two or more dif-

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ferent operations, the proper Table to use is designated at the end of the section describing that operation, as well as remarks on the proper use of the Table and any other notes which may serve to make the Tables of the greatest possible service.

2—Grading Over Extended Areas.

To illustrate this form of measurement, let us suppose that the Area enclosed within the square in Fig. 1 of Plate X to be the plan of a square lot drawn to a scale of say 1 inch to 10 ft., and that it is required to know the number of cu. yds. of material necessary to be removed to grade the lot so that its final surface shall be level and at a determined elevation.

Preparation of Diagram.

In the diagram let the elevation marked *Grade* be the final height to which the surface of the lot is to be brought, and let the lines marked 2.0, 4.0, 6.0, etc., be counter lines of the original surface or lines of intersection of the original surface of the ground with equidistant, parallel, horizontal planes—the cutting planes being two feet apart vertically—and the numbers at the end of each contour being the number of feet that the given plane is above the Zero or Grade plane to which the surface of the lot is to be brought by excavating.

It is evident that by passing these parallel, horizontal planes in the manner described we have divided the total volume of material necessary to be excavated to bring the surface of the lot down to the desired surface into ten continuous prismoids, having for their respective bases the areas included within the contour lines and the intercepted portions of the two sides of the lot, and for their lengths the vertical distance between the horizontal cutting planes, which in this case has been taken as two feet.

As we have assumed the excavation to be vertical at the sides of the lot, we have as the upper base of our first prismoid the area included between the Contour line 20.0 and the intercepted portions AB and AC of the sides of the lot, while the lower base of the prismoid is the Area included within the Contour line 18.0 and the intercepted lengths AD and AE of the sides.

The second prismoid has the area ADE for its upper base and the area AFG for the lower base, and so on to the last prismoid, whose upper base is evidently the area AMNV and whose lower base is the area ARSTV: the length of each prismoid being equal to the vertical distance between the cutting planes, is of course two feet.

Since these prismoids are contiunuous, the lower base of one prismoid being also the upper base of the one below it, it is evident that we can apply to it the same method of measurement as explained in our previous discussion of Railroad excavation under the the head of "Continuous Prismoids." To do this in this case, two adjoining prismoids, each having a length of two feet, are considered as a single prismoid having a length of four feet. The upper base of one prismoid and the lower base of the other prismoid are thus taken as the even As of Eqs, 14 and 15 Pg. 66 while the intermediate As are evidently the odd As of the same Equation.

In the particular example we are discussing, since there are 10 prismoids each 2 ft. in length, the total volume would be considered as the volume of 5 prismoids each having a length of 4 ft., and the demonstration would be exactly the same as in the previous case, and hence does not require further repetition here.

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Application of the Prismoidal Formula.

The general formula for the value of any number of continuous prismoids has been shown to be

$$V \text{ cu. ft.} = \frac{2 \times L}{3}\left(A_0 + 2A_1 + A_2 + 2A_3 + A_4 + \ldots + 2A_{n-1} + A_n - \frac{Ao + An}{2}\right) \quad \ldots\ldots(19)$$

or

$$V \text{ cu. yds.} = \frac{2 \times L}{3 \times 27}\left(Ao + 2A_1 + A_2 + 2A_3 + A_4 + \ldots + A_{n-1} + A_n - \frac{Ao + An}{2}\right) \quad \ldots\ldots(20)$$

In the particular problem we are using for illustration there are 10 small constituent prismoids each 2 ft. long.

In applying Eq. 19 to this particular case it is evident that the areas ABC and ARSTV of Fig. 1 Plate X are the A_1 and A_n of that Equation and since there are 10 of these small prismoids each 2 ft. long n will be 10 and L = 2.0

Designating the areas of the bases of these 10 constituent prismoids by the numbers of the contours on the diagram we have the areas 18, 14, 10, 6, 2 as the values of the quantities A_1, A_3, A_5, A_7 and A_9, respectively or the odd A's of Eq. 13, while the areas 20, 16, 12, 8, 4 and 0 are the corresponding values of A_0, A_2, A_4, A_6, A_8 and A_{10} or the *even* A's of that Equation.

Substituting all these values in Eq. 13 we have as the total volume of excavation

$$V = \frac{2 \times 2}{3 \times 27}\left((20) + 2\,(18) + (16) + 2\,(14) + (12) + 2\,(10) + (8) + 2\,(6) + (4) + 2\,(2) + (0) - \frac{(0) + (20)}{2}\right) \quad \ldots\ldots(a)$$

or generally

$$V = \frac{2 \times 2}{3 \times 27}\left(\text{Sum Even As} + 2 \text{ Sum Odd As} - \frac{\text{Sum Extreme As}}{2}\right) \quad \ldots(b)$$

Since our diagram is plotted to a scale of 1″ = 10′, 1 Sq. Inch = 10^2 = 100 sq. Ft., which being introduced into Eq. 13 gives for the total volume after reduction.

$$V = 4.9282\left(\text{Sum Even A's} + 2 \text{ Sum Odd As} - \frac{\text{Sum Extreme A's}}{2}\right) \quad \ldots\ldots(c)$$

By assuming the expression within the parenthesis of Eq. (c) to be 1 Sq. inch of actual area we have

$$V = 4.9382 \text{ Cu. Yds} \quad \ldots\ldots(d)$$

Eq. (d) shows that after having performed all of the operations indicated by the expression within the parenthesis of Eq. (a), each square inch of actual area indicated by the result of those operations when plotted to a scale of 10 ft. to 1 inch will represent a volume of 4.9382 Cu. Yds.

Method of Measurement.

Hence to measure the total volume of all the prismoids into which the total quantity of material to be excavated has been divided, we adjust the

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Planimeter to that Setting which will give a Reading of 4938.2 Vernier Units for each sq. inch of actual area traced. The areas ABC and ARSTV are then traced and one-half the resulting number of Vernier Units recorded is set down as a partial result.

The Planimeter is then adjusted to a zero Reading and all the bases of the constituent prismoids are traced—those bases designated as odd being traced twice and those designated as even being traced once. At the completion of this last tracing which is continuous and without intermediate readings, the total Reading is taken and from it is subtracted the partial Reading already recorded. The resulting number of Vernier Units is then multiplied by the relative value of the Vernier Unit for the scale in question, which in this case is .001, and the product thus obtained is the total value of the 10 prismoids or the total number of Cu. Yds. of material which will have to be excavated to bring the surface of the given lot to the required surface.

In the description just given of the use of the Polar Planimeter in the measurement of quantities of materials in grading and similar operations, the example chosen to illustrate this method of measurement while of very frequent occurrence was purposely made of very simple character in order that the method should be clearly understood, but the extension of the method and its application to much more complicated conditions is quite as simple and involves no principle other than those already described.

It will take but little thought to discover how easily this form of measurement can be extended to cover *every* form of operation similar to the one above described, no matter how complicated it may be and how perfectly the Planimeter is adapted to the solution of all problems of similar nature.

In the case of grading and leveling operations the Planimeter when used with any topographical map and by the simple operation of tracing the contours will give at once not only the volume of material necessary to be excavated to bring the surface of the ground to any desired level but also the amount of material necessary to fill in any depressions, thus enabling the quantities for cutting and filing to be compared and greatly facilitating the entire operation.

The facility and ease with which the Planimeter measures the volumes of materials in all these forms of operation has also the very great advantage of allowing cross-sections or contours to be taken much closer together than is customary and hence giving a much more accurate and satisfactory result, since by the use of the Planimeter we are able to measure at least three prismoids in the same or less time than it takes to measure one by any other method and with a far greater degree of accuracy.

As already stated the extension of this method of measurement to other problems of like nature is evident and will not require further description except to say that the value of the Polar Planimeter in every such problem will be found to be quite as great as is apparent in the simple case we have been discussing—a remark equally applicable to each and every application possible to the instrument.

Table No. 9 gives the Settings, Constants and other factors for adjusting the Planimeter to this form of measurement and for all the usual scales.

The factors there given are calculated in all cases on the assumption that the vertical distance between the Contours or End Sections is one foot which would make L = 1.0 ft. in Eq. 19. The correction applied to

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the final result for any other value of L is evident and consists simply of multiplying the result obtained by using the factors given in the Tables by the vertical distance in ft. between the parallel cutting planes. Thus in the example just given the results obtained by using the factors in the Table would be the total volume of all the prismoids if the contours were one ft. apart. Since the cutting planes are 2 ft. apart the result thus found must be multiplied by 2 to obtain the correct volume for this particular case.

3—Contents and Areas of Reservoirs.

There is perhaps no one problem in which the wide range of application and adaptability of the Polar Planimeter to the many forms of Engineering computation is better illustrated than in the following and were the capacity of the instrument limited to the one problem about to be described, it would still remain by far the most valuable of all the Engineer's assistants.

Description of Diagram.

In Fig. 2 of Plate X is shown in plan the site of a Reservoir and Dam on which the Engineer is required to make a report and estimate of cost.

The diagram shows the location of the dam and contour lines of the proposed reservoir, the zero or datum line being taken as the height of the water in the creek at the site of the dam and the contours show the water line of the Reservoir for each two feet in height of the dam above the datum, the number of the contour being its vertical elevation in feet above the zero plane.

We will suppose the diagram to be drawn to a scale of say 20 ft. to one inch and that the Reservoir is required to hold a given number of thousand gallons.

It is evident that the first step in the solution of the problem will be to determine to what height or contour the water must be raised in order that the Reservoir shall have a capacity of the desired number of thousand gallons. It is at once seen that this operation is exactly the same as that already explained for finding the volume of excavation from original contours since, by passing the horizontal parallel planes whose intersection with the surface of the ground form the contour lines, we have divided up the capacity of the Reservoir into a number of continuous vertical prismoids whose bases are the areas included within the respective contours and whose common length is the vertical distance between the equidistant cutting planes, in this case being one foot.

Application of Prismoidal Formula.

The formula previously demonstrated for finding the volume of any number of continuous prismoids of the form under discussion and given on Pg. 76 of this Chapter is

$$\text{V cu. ft.} = \frac{L \times 2}{3}\left(A_0 + 2A_1 + A_2 + 2A_3 + \text{----} + 2A_{n-1} + A_n - \frac{Ao \times An}{2}\right)\text{----(19)}$$

or

$$V = \frac{L \times 2}{3}\left(\text{Sum Even A's} + 2\,\text{Sum Odd A's} - \tfrac{1}{2}\,\text{Sum Extreme As}\right)\text{---(19)}$$

in which V was the total volume in Cu. Ft. of the n continuous prismoids

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and n is an even number. Since the vertical distance between the contours is one ft.

L = 1 in. Eq. 19 and since 1 U. S. Gallon = .13368 Cu. Feet by substituting these values in Eq. 19 we have

$$V = \frac{1 \times 2}{3 \times .13368}\left(\text{Sum Even As} + 2\,\text{Sum Odd As} - \tfrac{1}{2}\,\text{Sum Extreme As}\right) \text{--(21)}$$

or by reduction the Vol. in U. S. Gallons is

V = 4,987 (Sum Even As + 2 Sum Odd As — 1/2 Sum Extreme As) ---- (22)

which is the general equation required.

Since the given diagram has been plotted to a scale of 1 inch = 20 ft., 1 Sq. inch = 400 Sq. ft. and including this in the general formula we have as the Equation for this particular case (since 4.987 × 400 = 1994.8).

V = 1994.8 (Sum Even As + 2 Sum Odd As — ½ Sum Extreme As) (22) the required volume being expressed in U. S. Gallons.

Method of Measurement.

If then we adjust the Planimeter to that length of Tracer Arm which will give a Reading of 1994.8 Vernier Units for each Sq. inch of Actual Area represented by the values within the parenthesis, and then trace the various As in the manner indicated, it is evident that the resultant Reading due to the entire operation will be the volume in U. S. Gallons of the Contents of the Reservoir if filled up to the Contour selected as the A_n of the trial measurement.

For the reasons already given elsewhere, exactly the same result is obtained by using another length of Tracer Arm or Setting and multiplying the resultant Reading by a corresponding value of the Vernier Unit, the Setting and Vernier Unit selected as best adapted to the particular case being given in Table 12 opposite the given scale.

Looking in Table 12 we find the Setting for the scale 1 in. = 20 ft. to be 39.2 with the value of the Relative Vernier Unit. 30.0. Adjusting the Planimeter to this Setting and bringing the Instrument to a Zero Reading the Planimeter is then placed on the diagram in its most favorable p osition and the Tracer brought to the point selected for the point of beginning of the tracing.

It is evident that since we desire to find that Contour at which the surface of the water must stand in order that the Reservoir shall contain the desired number of gallons, this can only be done by trial, varying the number of prismoids or contours measured until we find that value of n which will give us the desired value of V in Eq. 22.

Since n must be an even number the first trial measurement is made by taking any even Contour as the An of Eq. 22, and the volume of all the prismoids, or its equal the Contents in Gallons of the Reservoir where the water line is the n Contour, is found as indicated by the Formula by trac ing continuously the Contours marked Ao and the An selected for this trial measurement. This Reading which when divided by 2 is the "1/2 Sum of Extreme As" of our formula is then recorded as a partial result.

Adjusting the Planimeter again to a zero Reading, each Even A up to the Contour selected as the An of this trial measurement is then traced once and each odd A twice—the tracing as in the previous measurements being continuous. At the completion of this tracing the total Reading is taken and from it is subtracted the partial result already recorded. The

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result when multiplied by the value given for the Relative Vernier Unit for the given scale in Table 12 is then the total number of Gallons which the Reservoir would contain, were it filled with water up to the level of the Contour taken as the An of our trial measurement.

It is evident that the result of this trial measurement will at once show whether the Reservoir when filled to the Contour chosen for the An of the trial measurement holds less or more than the number of Gallons decided upon and will act as a guide in selecting another Contour for the second trial measurement. In most instances the results of a second trial measurement, which is of course made in exactly the same manner as described for the first trial, will be sufficient to definitely determine the water line of the Reservoir when holding the number of Gallons of water previously decided upon.

The desired contour having been determined in the manner just described, the next step is evidently to find the number of acres of land covered by water when the Reservoir is filled to the height thus found.

Measurement of Area.

This is at once found by adjusting the Planimeter to the Setting given in Table 5 for the scale of the diagram, or 1 inch = 20 ft., and tracing the area included within the Contour found by the above operation. The Reading of the Planimeter, after tracing the given contour, when multiplied by the value of the Relative Vernier Unit given in the Table for the given scale, gives at once the number of acres of land covered by water and required for the given Reservoir.

Since the finding of the Water line as above described also determines the height, and hence the Crose-section of the dam required, the volume of the various materials entering into the construction of the dam is also at once found by adjusting the Planimeter to the Setting required by the given conditions and tracing the cross-section of the designed dam.

In fact, we could quite as easily cause the Instrument to give the estimated cost of the dam were it desirable to do so.

The Engineer who has made the above indicated calculations by any other method than by the aid of the Planimeter in the manner just described, will never again willingly revert to the old methods or believe that too much can be claimed for the value for the aid thus rendered by the new.

Obviously all that has been said as to the accuracy of the Instrument in the operations previously explained will apply with equal truth to that just described and need not be repeated.

In Table No. 12, under the heading of "Contents of Reservoir," is given the Settings, Constants, Vernier Units and other factors for all the scales commonly used in calculations of this nature. The derivation of these factors will be easily understood from the practical example we have used for illustration in the above explanation and from the general demonstration of the principle of the Planimeter given in Chap. III.

4—Displacement Diagrams.

Description of Problem.

The extension of the use of the Polar Planimeter to many other Engineering problems of similar nature to those already described can easily be made in the light of the explanations given and need not be further discussed here. There is, however, a certain class of problem more especially met with in the work of the marine engineer or constructor, and to which the peculiar principles governing the operation of the Planimeter

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seem to especially adapt the instrument. To this class belong those operations involviug the measurement of tonnage, the displacement of vessels, obtaining the coefficients of Water Lines, areas of wetted surfaces and similar applications too numerous to mention.

The special adaptability of the Polar Planimeter to all calculatious of this nature is at once apparent, as they will be readily seen to be in reality only special forms of the particular form of measurement we have been discussing. The use of the Planimeter, both in theory and practice, in all operations of this class will be readily seen to be alike in all, being modified only in details to meet special conditions, and the general application being clearly understood, the modifications necessary to meet special conditions can be easily and intelligently made.

As a typical problem of this class we will take the measurement of volumes from displacement diagrams and briefly describe the use of the Planimeter in this connection.

Preparation of Diagram.

As a requisite in this form of measurement an accurate displacement diagram of the given vessel or scow must be made, being carefully plotted to some suitable scale. To prepare this displacement diagram, the vertical distance or depth that the vessel or scow sinks when loaded, as compared with its depth when empty, is first found and then accurately divided into any number of equal parts.

The wetted perimeter of the vessel at each one of these equidistant parts must then be measured and accurately plotted to a suitable scale, each such area being superimposed on the others on the diagram. In the case of a scow, which is usually rectangular in plan, these displacement areas can be found by actual measurement, while in the case of a curved or shaped vessel they are more easily obtained from the builder's plans.

It will be readily seen that these displacements or wetted areas are exactly the same as obtained in our previous illustrations by the passing of the equidistant, parallel, horizontal planes through the mass of material to be measured, and that the entire operation is exactly the same as already described in the previous cases referred to. It is also evident that the method of measuring the volume of water displaced is identical with the method of measurement employed in the previous examples.

Method of Measurement.

The successive displacement areas being numbered in the manner already described the volume of displacement to any given depth in Cu. Yds. of water displaced is evidently found by Eq. 15, or

$$V = \frac{L \times D^2 \times 2}{3 \times 27} \text{ (Sum Even As} + 2 \text{ Sum Odd As} - \tfrac{1}{2} \text{ Sum Extreme As)} \quad \text{(23)}$$

where L is the unit vertical distance between the displacement surfaces and D is the lineal scale of the plotted diagram.

The volume of water displaced being thus found the corresponding volume of loaded material, is found by introducing the proper coefficient for the given material.

This coefficient is the reciprocal of the specific gravity of the material, and is found by trial for any given material by loading the ganged scow with a measured volume of the given material.

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As the conditions are the same, Table No. 9 will give the Settings and other factors for all problems or operations of this class when the units of measurement are the same. As the vertical distances between the displacement surfaces are usually taken as less than 1 ft., which is the distance adopted for the calculation of the Table, the final volume of displacement must be multiplied in any given case by the rates or per cent. which the particular interval in any given case bears to one foot, the Tabular interval.

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CHAPTER VIII.

Quantities of Materials (Contd.)

III. VOLUMES FROM ORIGINAL AND FINAL CONTOURS.

1. General Method of Measurement.

We have thus far in the discussion of the measurement of volumes with the Polar Planimeter assumed that the volume of material measured was simply material to be excavated or moved, and without reference to any particular design or purpose in its removal other than that the area from which the material was removed should be left as a level surface.

It is also seen that, as in the case of the measurement of the volume of material for which Fig. 1 of Plate X was used as an illustration, the volume was obtained from measurement of the Original or surface contours only.

It very frequently happens, however, that either we desire the surface of the ground after the material measured has been excavated or removed to conform to some given grade or shape, or that the excavation or removal of the material should be done in furtherance of some definite final plan; in both cases the problem then is to measure the volnme of material necessary to be removed in order to bring the original surface of the ground to the definite final surface or form.

The measurement of the volumes of material to be moved under these conditions can be very much simplified and shortened by the use of a specially prepared diagram which shows not only the contours of the original surface, but also the final contours or contours of the final surface to which we wish to bring the original surface either by excavation, by embankment or filling, or by a combination of both operations; by means of such a diagram the volumes of materials involved can be measured by one operation of the Planimeter, and the highest degree of accuracy attained with a minimum expenditure of time, labor and mental effort.

To illustrate the preparation of these diagrams and the method of measurement of the volumes involved with the Planimeter, two practical and frequently occurring examples have been taken, which will not only clearly illustrate the principles and methods employed, but will serve as examples of the application of the Planimeter to every form of problem of similar nature.

2. GRADING OVER EXTENDED AREAS.

Preparation of Diagram.

As the first example of this form of measurement, let us suppose the diagram on Plate VII to be the map of a lot drawn to a scale of say 1 in. = 10 ft. In this diagram let the curved full lines be contours of the original surface of the ground made by passing a number of equidistant horizontal planes, the lowest or datum plane being taken at the height of the lowest point of the surface marked 0.0 on the diagram: the vertical distance between the contours being 2.0 feet, and the height of any contour

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above the lowest or datum contour to be given within the circle on that contour.

Suppose it is desired to excavate and remove the dirt or rock until the surface of the lot shall be brought to a given grade or surface which shall be level parallel to the front line, and shall have a regular up grade, so that the surface at the rear line of the lot shall be say 8 feet above the surface of the front line—the front line to be at the same elevation as the datum or zero line of the contours. As the lot is taken as being 80 feet square, the rise in grade of the final or finished surface is evidently one foot in ten.

The lot being 80 ft. deep, it is evident that if we divide the depth of the lot into 8 equal parts, and through each part draw straight lines parallel to the front line, that these lines thus drawn will be the contour of the final surface when graded in the manner described, and that the elevation of each final contour above the zero level will be given by the figures in the margin at the ends of each such contour line on the diagram.

The two sets of lines just described are evidently then simply the contours of the original and final surfaces respectively, of the lot, or the lines of intersection of the parallel cutting planes with the original and desired ground surfaces, observing, however, that in this particular case the contour interval of the final contours is 1 ft., while for the original contours it is 2 ft.

It is evident that at each point of intersection of the two sets of contours, since one contour gives the height of the original surface, and the other contour gives the height of the final surface of the ground at that particular point, if we subtract the height of one contour from the heights of the other intersecting contours, the difference will be the depth of material which must be excavated at that point in order to bring the surface from the original height to the final height.

If now, we make this subtraction at each such point of intersection, and write at each such point the number of feet given by such subtraction, by connecting all points of equal difference by dotted curved lines these dotted lines will evidently be lines of equal cutting, and at every point of each such dotted line the depth of cutting necessary to bring the original surface to the final surface will be the same.

A little consideration of the diagrams as thus constructed will show that these dotted lines or lines of equal cutting are also contours of the original surface, which may be considered as the horizontal projection of the lines of intersection of a series of equidistant parallel planes passed through the original surface and *parallel to the designed final surface*—these planes being spaced *one foot apart vertically.*

It is also seen that these planes have divided up the quantity of material to be excavated necessary to bring the original to the final surface into a number of continuous prismoids. The bases of the various prismoids are evidently the horizontal projections of the areas included within the various dotted lines and the sides of the lot, the excavation being vertical on all the sides of the lot.

Method of Measurement.

Beginning then with the area included within the dotted line marked 00 and the sides of the lot and making this the A_0 of Eq. 14, and giving each following base its proper subscript up to the dotted area 8.0, which is the A_n of the formula in this case; the successive As are traced in the same manner as in all the other cases of the measurement of Continuous

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prismoids previously explained and the final and total desired volume is obtained.

It is to be remembered that, as stated above, the various areas thus traced are not the *actual* areas cut out by the parallel cutting planes, since in this case the cutting planes are not horizontal, but are parallel to the final surface of the lot, but they are the *horizontal projections* of those areas, and the required volumes are equal to the *horizontal projections* of those areas thus cut out multiplied by the *vertical* distance between the cutting planes, which in this particular case is evidently one foot.

To simplify the explanation of the method of measurement just described a simple case was taken as an illustration of the method, the final surface being an inclined plane, but the extension of the method to much more complex final forms will be found to present no more serious difficulties, while the method itself will be found to relieve all similar forms of calculation which may be treated in like manner of a very large portion of the time and labor inseparably connected with such calculation when made by any other of the usual methods.

It is of course readily seen that Table No. 9 contains all the factors for this form of operation the only correction necessary in any case being that required for any value of L other than one foot as the length between sections which is the value assumed throughout in the calculation of the Table. The method of making such correction is too evident to need further description.

3—Volumes of Materials in Reservoir and Similar Construction.

The following example of the measurement of the volume of materials from Original and Final Contours by means of the Polar Planimeter has been selected owing to the fact that not only is it a problem of very common occurrence in the Engineer's practice but also because it so clearly illustrates the enormous value of the instrument in this particular form of measurement.

In the example just used to illustrate this use of the Planimeter the conditions were made of the simplest nature—the final surface being taken as a simple inclined plane—in order that the principles involved in both the preparation of the diagram and its measurement might be clearly understood. This knowledge having been acquired the extension of the principle to the case of *any* problem of similar nature and requiring similar form of treatment is easily accomplished regardless of the seeming complexity of any special set of conditions which might be involved in any given problem.

The theory and practical application is the same for *every* problem of the class under discussion regardless of the form or shape of the required final surface whose contours may be straight, curved, or irregular. In each case the diagram is constructed by plotting both sets of contours or their horizontal projections, original and final, one set being superimposed on the other and the elevations of both sets being referred to the same reference or datum plane. The total volume of the constituent prismoids into which this treatment divides the volume of material to be excavated is then measured by means of the method of Continuous Prismoids with the Planimeter in exactly the same manner as described in the example just described.

The factors given in Table 9 are to be used in all operations of this nature, proper correction being made to the final result when the contour interval in any given case varies from that employed in the calculation of the table which for convenience is taken as one foot.

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The facility and ease with which calculations of this class are made with the aid of the Polar Planimeter has in many cases a very direct result of the very greatest importance, and especially in the case of problems similar to that about to be described since, by allowing the making of a number of different measurements by this method in the time required to make one by any other method, we are able to make the calculations and obtain the results of a *number* of possible Reservoir locations and thus intelligently select the best location by a comparison of the results obtained, and that with no greater expenditure of time or mental effort than is ordinarilly given to one such computation.

The diagram shown on Plate VIII. shows the lines of a small storage Reservoir constructed some years ago by the writer for the Suburban Water Co., of Pittsburgh, Pa.

Description of Problem.

As will be seen the site selected was on a side hill having rather a steep slope, and the reservoir proper was constructed partly by excavating, partly by embankment, and measured 128 ft. by 148 ft. in size at the top of the basin.

The irregular curved lines shown on the diagram are contours of the original surface, the cutting planes being parallel and horizontal and having a perpendicular interval of 4 ft. The zero or datum line was taken at the line between the cut and fill portion of the construction marked oo and shown dotted on the diagram, the elevation of the various contours above or below this elevation is given in the margin at the end of each contour.

The top of the embankment portion being 10 ft. wide, this width was carried level around the Reservoir basin as shown, making in the excavation part of the reservoir a level strip 10 ft. wide between the top of the inner or basin slope and the bottom of the outer slope. The elevation of this level strip which is marked "GRADE" on the diagram was taken at the same height as the zero plane of the surface Contours.

The slopes of the finished work both embankment and excavation being all taken as having a ratio of 1-1/2 to 1, the straight broken lines on the diagram are the lines of intersection of the same set of cutting planes as produced the surface or original contours with the slopes of the final or finished Reservoir surfaces and are the "final contours" or contours of the final surface.

The problem then is to measure the volume of the material which must be excavated and filled in, in order to bring the original surface of the ground to the desired finished surface or in other words to find the number of Cu. yds. of excavation and embankment required in order to construct a reservoir of the size and shape shown on a site whose surface is shown by the original contours.

Preparation of Diagram.

On examination of the diagram constructed in the manner just described it is seen that it shows the two complete sets of contours—one set being the "Original" or Surface contours, and the other the "Final" or Contours of the finished reservoir—the two sets of contours being made by the same set of parallel cutting planes and one set being drawn superimposed upon the other.

It will also be seen that by passing the cutting planes in the manner described we have divided all of the material necessary to be excavated

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above the Zero plane into a series of oblique continuous prismoids, the bases of the various prismoids being the respective areas enclosed by any two intersecting contours, one in each set of contours having the same elevation above the datum plane, and the length of each such prismoid being the *vertical* distance between the cutting planes, which, in this instance, is four feet.

Thus, in the diagram the first prismoid would have for its upper base the area enclosed between the contour numbered 36 of the "Original Contours" and the contour number 36 of the "Final Contours" or the area included between the curved line marked 36 and the broken line A′ B′; the lower base of the prismoid being the area included between the contours marked 32 of the two sets of contours, or the area included between the original contour 32 and the broken line C′ D′, and so on to the last prismoid whose upper base would be the area OVUT and whose lower base would be the area PSML.

Since the dotted line O-O or PS is the dividing line between the excavation and embankment portions of the Reservoir, the total volume of embankment is evidently the total volume of the prismoids into which the parallel cutting planes have divided that portion of the Reservoir *below* the level assumed as grade. The bases of these prismoids being formed in exactly the same manner as those *above* the O-O line just described, these prismoids need no further description since they differ only from the other prismoids in that they are the volumes of embankment necessary to bring the original surface up to the required surface instead of volumes of excavation.

Method of Measurement.

It is at once evident that the measurement of the volumes of these prismoids with the Planimeter is accomplished in exactly the same manner as described in detail in the examples already given for the measurement of continuous prismoids, and that the settings and other factors to be employed are the same and are those given in Table 9, remembering always to make the proper correction in cases where the contour interval differs from that used in the table mentioned.

The particular case we are using for illustration would evidently require three distinct sets of measurement operations; the first operation would give the volume of excavation *above* the level or elevation taken as grade; the second would be to find the volume of excavation in the basin proper *below* the grade level, the result of which being added to the first result would give the total number of cubic yards of material to be excavated; while the third operation would be for finding the number of cubic yards of embankment required for the given construction.

With the examples already given of the use of the Polar Planimeter in the measurement of volumes of prismoids, no difficulty will be encountered in applying the instrument to the measurement of *any* quantity which can be treated by any of the methods described, and with the principles involved clearly understood there is almost no limit to the number or variety of problems to the solution of which the Planimeter cannot be most easily adapted and with the same degree of accuracy and almost incredible saving in time, labor and mental effort which characterizes its use in those applications already discussed.

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CHAPTER IX.

Quantities of Materials (Cont'd.)

IV.—Special Methods of Measurement of Quantities.

As already stated, those principles of the Polar Planimeter which make the instrument of such great value in the various forms of measurement already described are also such as to make the instrument almost universal in its application to all forms of engineering computations. In fact, there are very few of the calculations so constantly met with in engineering practice in which the aid of which the Planimeter is capable is not as efficient and valuable as in those selected for illustration. The simple enumeration of these practical applications would extend the limits of our discussion far beyond the allotted space and will not be attempted, but a brief description is desirable of a few of the more important of these in order to clearly explain the use of the factors given in the tables for the operations in question.

When not definitely stated the table to be used in any given case is easily determined by the conditions of the given problem. In many cases the selection of the table from which to take the factors for adjusting the Planimeter is determined by the unit in which it is desired that the results should be expressed. For example, in making the measurement for the volume of contents of a reservoir if the result is desired in U. S. gallons the setting would be taken for the given scale from Table 12, while if the result is to be expressed in cubic feet Table 9 would be used.

The tables in all cases are made very complete so that the accuracy of results can be very readily checked, usually by other factors given in the same table.

1. Volumes of Brickwork.

In most instances of the use of brick in engineering construction work the amount of brickwork is estimated and paid for at a given price per thousand brick "laid" or "in the work." In a great many cases the operation of determining the number of brick used in any given work is a very uncertain and often unsatisfactory one due to the irregular shape of the brickwork and its varying quantity in different portions of the work.

This is especially true in the case of oval or irregular shaped brick sewers where the very nature of the material excavated may cause a most irregular shape of cross-section, or in case of a tunnel such as shown in Fig. 6 of Plate V, where the cross-section of the brick lining would be constantly varying and would require measurements at near intervals—often a very difficult and almost impossible operation by ordinary methods.

The ability of the Planimeter to accurately measure the area of any figure regardless of the irregularity of its outline, and to give the measurement of that area in any desired unit and for any scale of drawing makes the instrument of very great value in this connection.

In table 11 are given the Settings, Vernier Units and other factors for adjusting the Planimeter for use in the measurement of brickwork for all of the commonly used scales.

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Method of Measurement.

To illustrate the use of the table in this connection, let us suppose Fig. 6 of Plate V to be the cross-section of a tunnel to be lined with brick in the manner shown, the irregularities of the tunnel to be filled in with brickwork, and that we desire to know the number of brick used per linear foot, the drawing being made to a scale of 1/4 inch to 1 foot.

Looking in Table 10 we find the setting for the given scale to be 285.6 and the value of the Relative Vernier Unit to be 4.0.

Adjusting the Planimeter to the given setting, and having brought the instrument to a Zero reading the area of the cross-section of the brick lining is traced in the usual manner. Let us suppose the reading of the instrument for this tracing to be say 1250 Vernier Units. Then since the value of the Vernier Unit for this scale is 4.0 we have

$$9256 \times 4 = 37024$$

which is the number of brick per linear foot required.

It is evident that since the Planimeter gives the number of brick per linear foot for the area traced, the quantities of brick in house walls and similar forms of construction is at once obtained in exactly the same manner by simply tracing the elevation of the wall directly on the plan of the building and multiplying the result by the required correction for any given thickness of wall other than 1 foot. The openings for door or windows can be traced and the results substracted in instances where such correction is required but ordinarily this is not done in the measurement of house walls, the walls being usually counted as solid to allow for the extra work involved in the construction of such openings.

If, as is often the case and especially in retaining walls or foundations the result is required in Cu. Ft. or in Cu. Yds. instead of number of brick, Table 3 or Table 3[1] would be used respectively in place of Table 11—the operation however being exactly the same in each case whichever Table were employed.

In the preparation of the Table the average number of brick per Cu. Ft. was taken as 18.5 brick which will be found to be a very close approximation, and any result obtained by the use of Table II can be at once reduced to Cu. Ft. or Cu. Yds. by a simple multiplication.

The method of finding the factors given in this Table is easily seen from the general Equations and mathematical discussions given in Chapter I and need not be repeated here. The Table is intended to cover every possible case in the measurement of brickwork and factors are given for every scale to which plans for brickwork are usually drawn.

2. Weights of Metals.

The ability of the Polar Planimeter to give any desired Reading for any given actual area circumscribed by the Tracer admits also of the use of the instrument in many operations to which it otherwise would not be applicable and this principle of the Planimeter can often be made use of to very great advantage.

It very often happens that it is required to know the weight of a piece of metal which, from the irregularity of its shape, the absence of the proper tables, or some other reason makes the necessary calculation either impossible or at best a long and tedious operation.

It will take but little consideration to see how the principle of the Planimeter just mentioned can be utilized in a case of this kind by so adjusting the instrument as to cause it to record the desired weight of the metal when the tracer has been passed about its plotted boundary.

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Method of Measurement.

To illustrate this application let us suppose we have a piece of iron of the shape shown in Fig. 4 of Plate 4. the total weight of which is required and that the given figure is an accurate drawing of the article drawn to any convenient scale—say ½ inch to 1 foot.

Since the scale of the given drawing is ½ inch to 1 foot, one actual square inch of the drawn figure will represent an area of 4 square feet of metal.

Since the material is Iron and 1 sq. ft. of Iron taken as 1 inch thick weighs 40.0 lbs., and since 1 sq. inch of actual area of the drawing represents an area of 4 sq. ft., the weight represented by 1 sq. inch of actual area will be $40 \times 4 = 160$ lbs., and the total weight of the article will be 160 lbs., taken as many times as there are square inches of actual area in the drawing of the figure.

If then we adjust the Planimeter to that Setting which will give a Reading of 1600 Vernier Units for each square inch of actual area of figure traced, it is evident that the Reading due to tracing the given figure will be ten times its weight in lbs., and to obtain the weight of any piece of flat iron drawn to a scale of ½ inch to 1 foot we trace the outline of the given figure with the Planimeter adjusted as described and the resultant Reading divided by 10 or multiplied by .1 will be the desired weight in lbs. assuming the iron to be 1 inch thick. Multiplying this result by the thickness of the iron in inches will evidently give the total weight required.

Table No. 11 has been calculated in the manner just described and gives the Settings, Vernier Units and other factors for all the scales used in plotting construction work or similar nature.

The table has been calculated on the assumption that the metal measured was 1 inch in thickness or length and the result of any tracing must be multiplied by the total thickness or length in inches of the metal measnred.

Gauge Points and Their Use.

At the foot of Table 12 will be found the names of several of the more common metals used in construction work together with the value of what is termed the Gauge Point of that metal. In all cases this value is the ratio between the weight of any material in the metal whose gauge point it is, and the weight of the same quantity in Iron ; the weight of the material being found in Iron by the use of the Table, its weight if made of any other metal is found by multiplying the weight in iron thus found by the Gauge Point of the desired metal.

For example, let us suppose the metal whose weight is desired to be brass whose Gauge Point as given is 1.09. The operation just described and with the factors given in Table 12 give the weight per linear inch of the given area if the material were iron, and to obtain its weight in brass we multiply its weight thus found in iron by 1.09 the Gauge Point given for brass—the product being the desired weight in brass.

The unit of length has been assumed in this form of measurement as one inch instead of one foot as probably covering more cases of actual problems than the foot unit, although in the cases of Beams, Girders, built up Sections, etc., the foot unit would oftenest apply.

Gauge Points of Various Metals.

Wrt. Iron—	1.00	Copper—	1.15
Cast Iron—	0.93	Brass—	1.09
Cast Steel—	1.02	Lead—	1.47
Steel Plates—	1.04	Zinc—	0.92

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CHAPTER X.

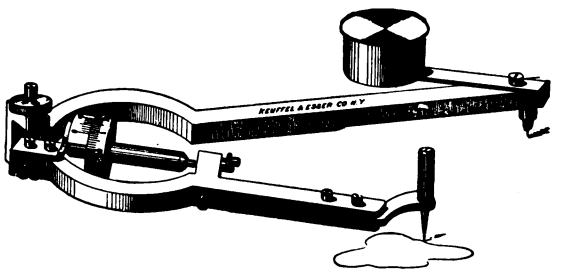
Various Forms of Planimeters and their Characteristics.

a. Area Planimeter.

As has previously been stated, there are many forms of the Planimeter varying from the simple Area Planimeter whose capacity is usually limited to a single operation, to the complicated Precision Instruments whose range of operation and attainable degree of accuracy is almost unlimited.

A brief description of these various forms of the instrument together with the operations possible to each and any particular application to which they may be especially adapted should be of interest here.

The simplest form of Planimeter consists of two arms of unchangeable length connected at the end by some form of pivot joint. The free end of one arm terminates in a vertical needle, which on being pressed into the paper, becomes a center about which the entire Instrument can revolve. At the free end of the other arm is the vertical Tracing Needle or Tracer which is used to follow the outline of the figure whose area is required.



No. 1.

Polar Planimeter, German Silver, to measure square inches.

At the other end of the Tracer Arm and near the pivot joint connecting the two arms is the Integrating Wheel having its axis parallel to the Tracer Arm to which it is attached.

The drum of the Integrating Wheel is divided into one hundred equal parts, and the relation of the areas and Wheel is such as to cause the Wheel to record one hundred of these parts, or to make one complete revolution when the Tracer has traced an area of ten square inches. By this arrangement the figures on the Wheel represent square inches, the intermediate graduations tenths, and the vernier hundredths of a square inch of area traced.

Having no recording wheel the limit of the area which can be traced and recorded is evidently ten square inches while the length of the Tracer

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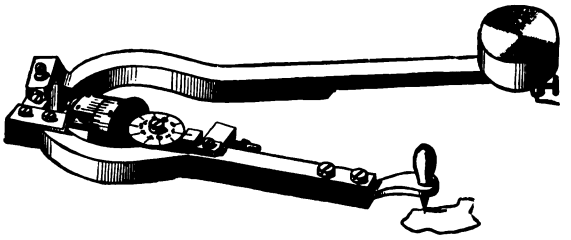
Arm being constant restricts the measurement to one unit of area for all measurements, usually one square inch.



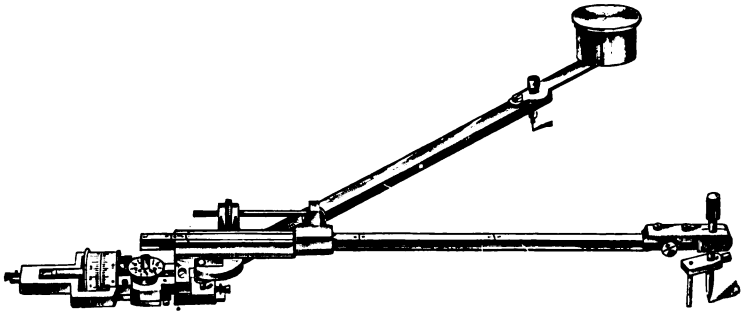
No. 2.

The addition of a recording wheel to this form of Planimeter which records the number of complete revolutions of the Wheel for any tracing allows of the measurement and recording of an area of one hundred square inches for those instruments having this attachment.

The simplicity of this form of Planimeter restricts its use to the simple operations of measuring areas within its capacity, although it performs that operation with a degree of accuracy unapproachable by any other method when applied to the measurement of areas having more or less irregular bounding lines.

b. Polar Planimeter.

The next higher form of Planimeter to the simple instrument just described differs from the simpler form in having a tracer arm whose length can be increased or diminished. Since any change in the length of the Tracer Arm produces a change in the number of units recorded for the tracing of any given area, this form of instrument admits of the adjustment of the tracer arm so as to give the area of any figure traced in more than one unit of area, and to allow of its use in measuring figures drawn to different scales.



No. 3.

Index marks are usually engraved upon the tracer arm to which the arm must be adjusted in order to give the area traced in terms of the unit of area desired. Thus, when set to one such mark the area recorded will be in square inches, in another square yards, in another square decimeters and so on, the unit given by any given index mark being engraved on the arm at that mark.

Owing both to the adjustable length of arm and the larger size of this form of Planimeter this instrument is capable of measuring much

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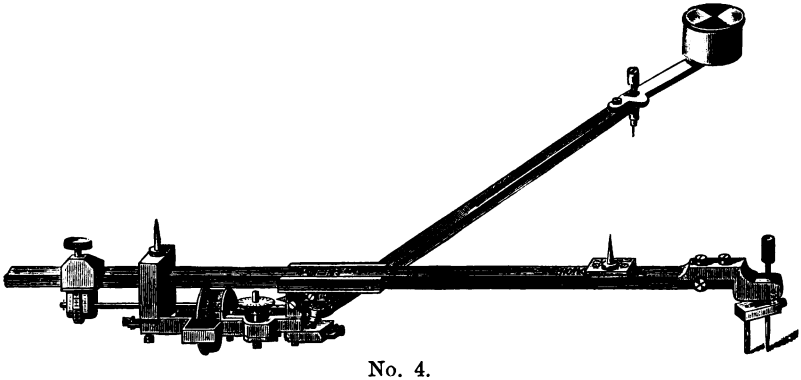
larger areas at one operation than is possible to the simpler form and with the same degree of accuracy, thus largely increasing its usefulness in all such operations.

While any of the forms of Planimeter so far referred to may be used to find the mean height of an indicator or similar diagram, the operation is evidently an indirect one and necessitates more or less mathematical computation to arrive at the desired results.

To obviate this a special form of the kind of Planimeter under discussion is made by means of which the Planimeter is enabled to give the average height of these diagrams, without having to submit the results to subsequent calculation.



No. 4.

This special form of the instrument has two conical needle point projections extending upward from the Planimeter. One of these projecting points is attached to the Frame or Carriage of the instrument while the other is attached to the top of the Tracer Arm and moves with it. By loosening the binding screws of the Tracer Arm and placing the Carriage Point at one end of the base of the diagram the Tracer Arm is slid through its bearings in the Carriage until the needle point on the top of the Arm is brought to the other end of the base of the diagram, in which position the Tracer Arm is firmly fastened by means of the clamping screws of the Carriage.

By this operation the distance between these two needle points is made equal to the length of the diagram, the mean height of which is to be measured. The points on the Planimeter at which these needle points are attached are so arranged that the instrument having been thus adjusted and the outline of the diagram traced, the resulting reading when multiplied by a certain Constant given for each instrument will be the average mean height of the diagram traced.

As this special attachment in no way interferes with the use of the Planimeter in any other of its applications, this attachment is one of very great convenience and importance where there is much use of the Planimeter in this form of operation.

c. Improved Polar Planimeter.

A still higher form of the Polar Planimeter is that having an adjustable tracer arm which is graduated for its entire length into some scale of equal parts, usually and preferably the value of one division of the graduation being ½ millimetre and which is sub-divided by the Carriage Vernier to read $\frac{1}{20}$ millimetres.

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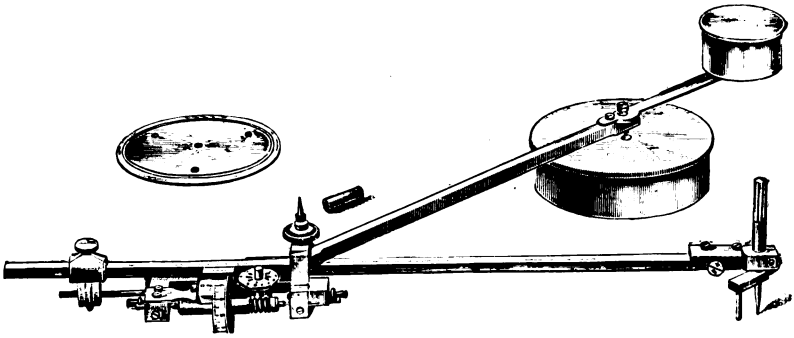
In this form of instrument the needle forming the Pole at the end of the Polar Arm is discarded in favor of the Ball Pole and weight which adds much to the facility and accuracy with which results are obtained in all its forms of operation.

As this is the form of Planimeter used throughout our entire discussion and described in detail in Chapter II. it needs no further reference here except to repeat and emphasize the statement already made, that of the many varying forms of Planimeters it is that one which is best adapted to the General uses of the Engineer, and is that instrument of all others, capable of rendering him invaluable aid in any and every form of his professional work.

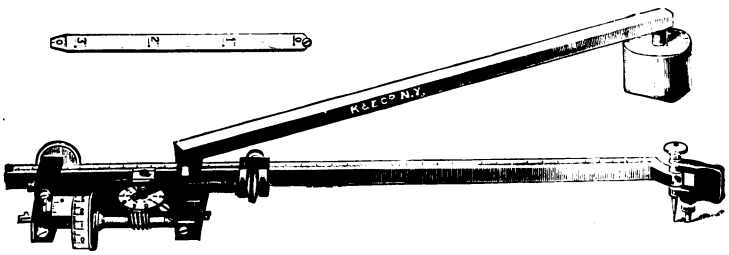


No. 5.

c. The Compensating Planimeter.

All that has been said either here or in previons chapters of the Polar Planimeter, just mentioned, can be repeated with even greater truth and force of that form of the instrument designed by Mr. G. Coradi of Zurich, and termed by him the "*Compensating Planimeter.*"

Reference to this statement has to be made in a previous Chapter and its advantages over other forms of Polar Planimeters due to the elimination from its results of those errors inherent in the ordinary Polar type are quite sufficient to place it within the Precision class.



No. 6.

Mr. Coradi regards this instrument as "destined little by little to replace the Polar Planimeter"—a prediction which the instrument seems fully to justify. The theoretical considerations involved in the design of the Compensating Planimeter are easily understood from the mathematical discussion of the principles of the Planimeter given in Chaps. II. and III. and the manner in which the tendency to error in the results of Plainimeter measurements due to non-parallelism of the axis of the Integrating

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Wheel and other causes are compensated for in the working of this form of instrument will be readily seen from the two diagrams given in Plate IX., Figs. 5 and 6.

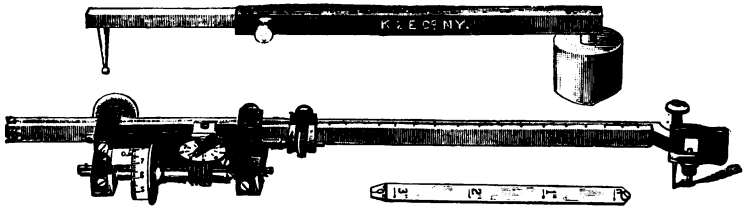
These Figs. show the Compensating Planimeter as placed in position for the measurement of the area J. As explained in Chap. III., when the axis of the Integrating Wheel in any Planimeter is not in adjustment or not parallel to the Tracer Arm, it produces an error in the results of any measurement, the size of the error being directly proportional to the amount of the displacement.

By taking the mean of two Readings, one taken with the Planimeter on the *right* of the base XX^1, as shown by the full lines in Figs. 5 and 6, and one Reading taken with the instrument on the *left* of the base as shown by the dotted lines in the given Figs., any error in the results due to the causes named is evidently eliminated, since in one position the sign of the error would be positive, and in the other negative. Detail drawings of the Compensating Planimeter are given on Plate IX. from which the construction of the instrument can be readily understood.

The Pole of the Planimeter is a very fine needle point projecting from the bottom of the Polar Weight P shown in Fig. 4. This Polar Weight is permanently attached to the Polar Arm and revolves with the Arm about the needle point as a center, the bottom of the Polar Weight being wedge shaped in order to reduce friction to a minimum.

The special arrangement by which the length of the Tracer Arm is made equal in length to the base of any diagram whose average height is to be measured has already been described on Page 55 of Chapter IV., as has also the special form of graduation by which the "Setting" of the Planimeter for any case is also the length of the Tracer Arm, an improvement of much value when making the calculations of the factors for any given operation.

The workmanship of this instrnment is all that could be desired and of the kind which characterizes all of this maker's productions.



No. 7.

A special form of the Compensating Planimeter has recently been produced which has an adjustable Polar Arm which may be shortened or lengthened as desired. By this arrangement the value of p in the Equation of the Constant Circle (Figs. 1 and 2 of Plate III.), can be made such that the Constant for any given scale or operation shall be a round number or any number desired.

A further advantage of the adjustable Polar Arm lies in the ability to extend it and thus increase the size of the Area which may be measured with the instrument.

As giving an idea of the actual size of the Figure whose area can be measurad in one operation of the Planimeter it can be said that the Compensating Planimeter as described above is capable of measuring with one tracing and with the Pole on the outside of the given figure a figure in

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the form of a square having a side 25 centimeters in length or an area of, say, 625 Sq. Centimeters, equivalent to about 97 Sq. inches, which is also about the same capacity as the Polar form of instrument which we have been using for illustration and description.

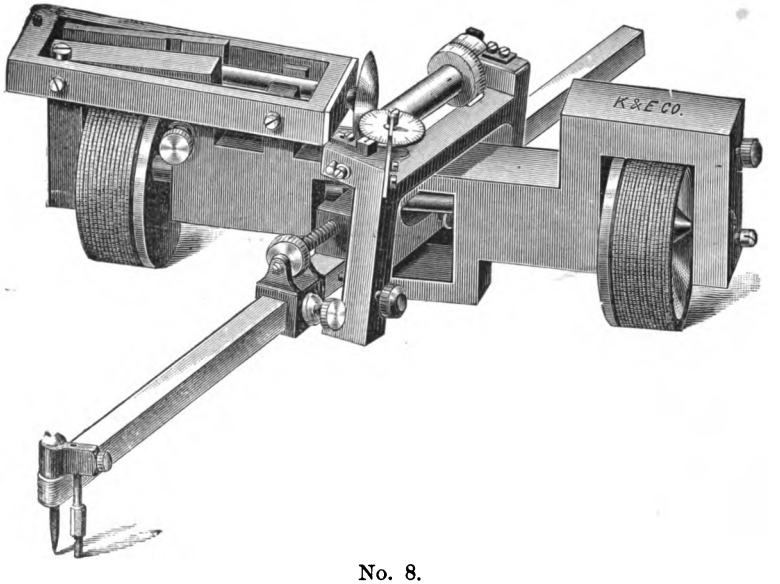
d. Precision Planimeters.

In all forms of the Planimeter thus far described the rotation of the Integrating Wheel has been due to frictional contact between the rim of the Wheel and the paper or other surface which it rolls.

The method by which any error in the results of measurements with the Polar Planimeter due to irregularities or imperfections in the paper upon which the wheel moves is miniminized, has already been explained in a previous chapter.

Attempts to entirely eliminate this source of error and make the accuracy of the instrument independent of the nature of the surface upon which the area to be measured is drawn, has resulted in the production of a class of instruments termed Precision Planimeters, in which this elimination has been successfully accomplished. The importance of these instruments makes a brief description of them desirable in this connection.

Precision Rolling Planimeter.



No. 8.

That form of Planimeter which may be considered as best representing the Precision class of instrument is what is termed the "Precision Rolling Planimeter" by its maker Mr. Coradi of Zurich detail drawings of which are given on Plate X.

It will readily be seen from the drawings and from the following description of the characteristic features of its construction that the accuracy of the instrument and its operation is entirely independeut of the nature or condition of the surface upon which it works, enabling measurements to be made with it of the areas of figures drawn on poor paper, tracing cloth or other material with the same degree of accuracy as though the surface were of the most favorable character.

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By referring to the drawing it is seen that the Planimeter has three points of support, the two wheels or rollers R and R′ and the tracer rest Z. These wheels or rollers are of exactly the same size and are rigidly attached to an axis which carries the frame and recording apparatus of the instrument. The bearing surfaces of these wheels are covered with fine slightly projecting points or teeth which entirely prevents any slipping whatever of the wheels on the surface over which they roll. The rigid connection of the wheels with the axis which turns with them prevents any movement of the Planimeter in any direction other than backwards or forwards along a straight line at right angles to the axis—this straight line being the "base" of the instrument and differing from the base of the Polar form, which it will be remembered is a circle.

One of these large wheels is provided with a fine toothed rack F in which engages the teeth of a smaller wheel E fixed upon the steel axis B, the end of the axis carrying the spherical segment M. The spherical segment M is of very finely hardened polished steel and when in use is lowered until it comes in contact with a polished steel cylinder C, to which it imparts its motion by frictional contact. To this steel cylinder is attached the recording mechanism of the Planimeter W, V and O which by means of the parts mentioned thus records the number of revolutions and fractions of a revolution of the wheels R′ due to any movement of the Planimeter in the direction of the base.

The Tracer Arm F is graduated for its entire length having the zero of its graduation at the tracing needle D. This arm slides in sleeves in the frame and can be adjusted to any given length or Setting by means of the Vernier P. When desired an extension to the Tracer Arm is furnished by means of which its length can be greatly increased, thus adding to the size of the area which can be measured with the instrument in one operation.

The range of the Tracer Arm is about 30 degrees on each side of the base or center line which with the usual length of Tracer Arm allows of the measurement of an area 50 centimeters in width and of unlimited length being performed in one operation—a capacity not possessed by any other form of Planimeter.

This instrument is made in two sizes—the smaller size having a length of Tracer Arm of 20 cms. and the larger of 30 cms., which lengths can be increased by means of the extension mentioned above to 40 cms. and 50 cms., respectively.

The high degree of accuracy of this Planimeter together with its capacity and independence of the nature of the surface on which it operates gives it the highest place in the class of instruments to which it belongs.

Precision Disk Planimeter.

Another form of instrument which, like the Rolling Planimeter just described, is included in the Precision Class on account of the high degree of accuracy of which it is capable, is the Precision Disk Planimeter.

With this instrument as with others of its class this high degree of accuracy is due to the elimination of those sources of error present in the simpler forms of Planimeter, chief of which being the irregular and slipping movements of the instrument caused by uneveness and lack of uniformity of the contact surface when the integrating wheel receives its motion from direct contact with the surface over which it moves.

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This Planimeter consists of two distinct parts—the Polar Disk P having a rim on which are cut very fine teeth and the Planimeter proper AFHSM.

The Tracer Arm F of this instrument is of the form common to all Planimeters and like them is adjustable in length and capable of being adjusted to any desired Setting, and has also the usual Tracer Rest, Binding and Slow Motion screws.

The Polar Arm is supported on the paper by the small wheel L and the Tracer Rest S, and on the Polar Disk P by means of the small spherical bearing at p, the exact center of the disk, and turns about a finely polished pivot at this point which forms the center of revolution or Pole of the instrument. Any movement of the Tracer f produces a revolution of the entire instrument about the point p and this movement being communicated to the disk S through the Wheel r causes a corresponding movement of the Integrating Wheel R which moves around the edge of the Disk S—the extent and nature of the movement of the Tracer f being thus measured and recorded by the wheels R and Z.

Since the Rest S and the small wheel L are mere supports of the Tracer Arm they have no effect on the movement of the instrument, and since the motion of the Integrating Wheel R is produced entirely from its contact with the Disk S the surface of which is uniform, the accuracy of the instrument is seen to be entirely independent of the character of the surface traced.

The diameter of the Polar Disk is about 15 cms. and the total length of the Tracer Arm 35 cms. while the area which can be traced from one position of the instrument would be that of a figure 25 cms. in height by 20 cms. in length or about 80 sq. inches.

Referring again to the relative degrees of accuracy of these Precision Planimeters as compared with the Polar form used throughout the preceding discussion to illustrate the various operations of which it is capable, it can be said that in most cases equally accurate work can be done with the Polar or Compensating Instruments as with the instruments included within the so-called Precision class, provided that in the use of the simpler instruments *the conditions for accuracy given in detail in a previous chapter are carefully complied with* and that the surface or paper upon which the integrating wheel rolls is of the nature proved by experience to be most favorable for accurate results. The special value of the Precision Planimeters being as already stated their independence of these conditions.

The class of instruments known as Mechanical Integraphs and Integrators while differing from the Planimeters already described, must still be included with them since they possess most, if not all, of the properties of Planimeters.

In addition to an ability to perform those operations of which the ordinary form of Planimeter is capable, the Integraph and Integrator have a range of operation impossible to the simpler instrnment and which makes them of the greatest interest and value besides rendering their aid almost indispensable in certain forms of calculation.

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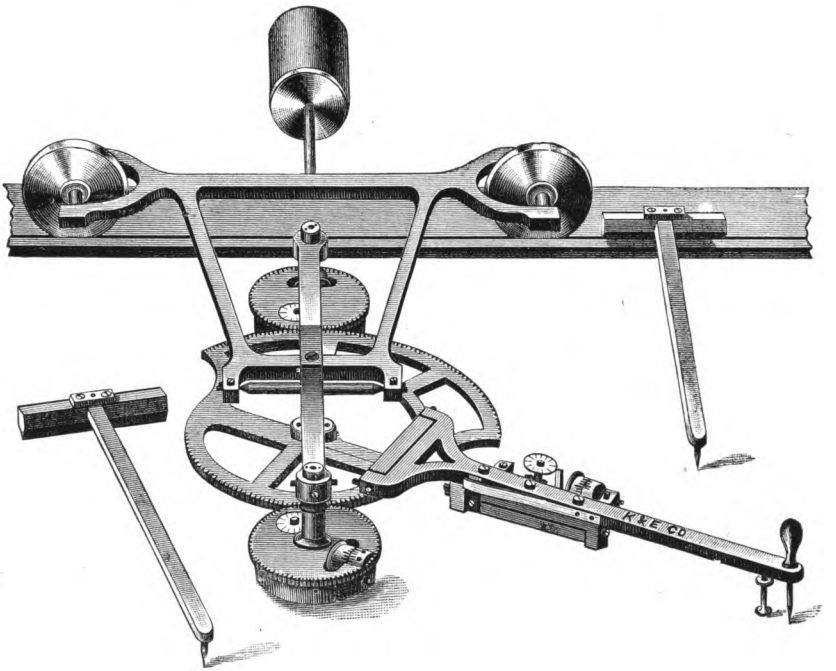

No. 9.

Lack of space prevents any adequate description either of the instruments themselves or of their characteristic properties other than to say that by their aid not only are moments, curves of stability and differential and integral curves, mechanically obtained and with the highest degree of accuracy, but in some instances—notably Coradis Integraph—*the instrument directly plots the curves themselves*, making the operation one of automatic graphical integration.

When to these operations are added the immediate mechanical calculation of moments of mertia and gyration, the solution of problems involving the finding of the Centers of gravity, of problems of stability and flotation—in fact of almost every operation occurring in the practice of the Bridge Engineer or Naval Architect the practical value of these instruments is apparent.

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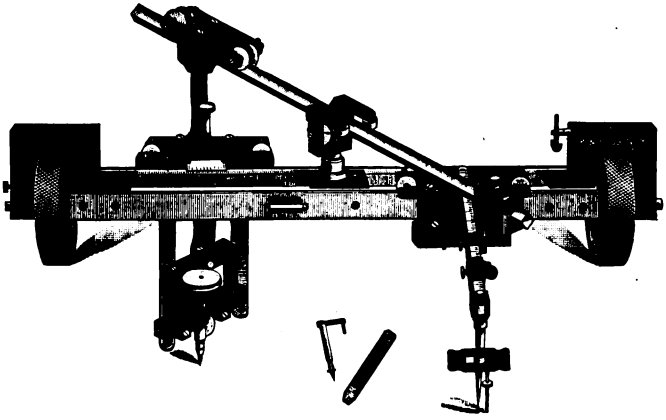
CORADI'S MECHANICAL INTEGRAPH.



No. 10.

Although lack of space prevents even the enumeration of many very valuable forms of instrument which from the nature of their theory and operation would be included in the Planimeter class we have been discussing, enough has been said to give some idea of their practical value and of the almost indispensible aid of which they are capable in almost every form of Engineering calculation.

That the value of these instruments is becoming slowly recognized is true, but the slowness of the recognition can only be accounted for by a lack of knowledge of the instruments and of their invaluable characteristics. With this knowledge and by the acceptance of the aid furnished by these instruments in almost every detail of Engineering labor, the work of the Engineer regardless of its nature will be relieved of by far the greatest share of its present attendent drudgery.

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CHAPTER XI.

Accuracy of Planimeter Measurements.

While the degree of accuracy in results attainable in the various operations of measurement with the Planimeter as described in the previous chapters has been given in the discussion of the use of the Planimeter in those descriptions, it will be of interest in conclusion and give a clearer understanding of the general subject of the accuracy of Planimeter measurement to give the results of the experiments which have been made at various times to determine the relative and actual degree of accuracy which may be confidently expected in the various forms of operation.

Not only is a knowledge of the degree of accuracy attainable in these operations of the greatest possible value as furnishing proof of the reliance which may be placed in all results of measurements thus obtained, but they serve the added purpose of showing the almost incalculable value of the aid these instruments are capable of rendering in every form of Engineering work and giving them the high place in the list of the Engineer mechanical assistants for which that accuracy and adaptability so eminently fits them.

While this subject has engaged the attention of a number of investigators, and many experiments have been made at times with more or less valuable results, perhaps the most valuable conclusions reached have been those due to a series of experiments conducted by Prof. Lorber while investigating the subject and reported by Prof. Shaw in his admirable paper read before the Institution of Civil Engineers of England and reprinted from their Proceedings.

In conducting these tests no attempt whatever was made to have the conditions during the experiments any more favorable than would be the case in the ordinary use of the instrument, so that the results obtained are those which may confidently be expected in every case in which the Planimeter is understandingly used and in compliance with the conditions for accuracy already given in a previous chapter.

It should be stated that the results obtained and reported were from a single tracing of the figure measured and not the average of a number of repetitions of the same measurement, and that the results are the average of nine different Planimeters, thus making them equivalent to the results of an average instrument.

From the series of experiments referred to Prof. Lorber concludes that "the different angles at which the measuring roller of the Polar Planimeter acts has little effect upon the results." He also found that taking one turn of the measuring roller, as + 100 Sq. Centimetres, the average error in the reading was only from 0.00075 to 0.0013, according as the center of rotation or the pole was without or within the area to be measured.

As the results of all of his experiments, Prof. Lorber offers the following empirical formula as giving the average error for the different instruments for average conditions:

$$d F_n = Kf + n \sqrt{Ff}$$

in which

n = Reading of Integrating Wheel
F = Actual Area to be measured
$d F_n$ = Error in result expressed in terms of the Area

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Applying to the various forms of Planimeters, of which we have given descriptions in the previous chapters, and introducing the values furnished by experiment for the factors k and n, we have as the error of the different classes of Planimeter

$dF = 0.00126\,f + 0.00022\,\sqrt{F\,f}$ for the Polar Planimeter

$dF = 0.00069\,f + 0.00018\,\sqrt{F\,f}$ for the Precision Polar Planimeter

$dF = 0.0009\,f + 0.0006\,\sqrt{F\,f}$ for the Rolling (Coradi) Planimeter

The degree of accuracy as determined by the application of these formulæ to a given operation may be inferred from a trial measurement taken with the Coradi Rolling Planimeter where $f = 100$, which gave the relative error of that instrument for the given instrument as

$$\frac{dF}{F} = \frac{1}{13330}$$

a degree of accuracy unattainable by any other method of measurement.

The actual results attained by the application of Prof. Lorber's formula is best shown in the following table, prepared by him as showing the results of his investigations:

TABLE

Actual Area Measured (F) c/m²	Absolute Error in Result (dF_D c/m²	Relative Error in Result f = 100 c/m²	Per cent.
200	0.157	1 in. 1274	0.0785
100	0.148	1 in. 682	0.148
50	0.141	1 in. 355	0.282
20	0.135	1 in. 148	0.675
10	0.133	1 in. 75	1.330
5	0,131	1 in. 39	2.620

The above results are from single, not repeated, tracings of the given area, and from the table it is seen that

1° Absolute Errors differ but little and are not proportional to the Actual Area traced.

2° The Relative Error diminishes rapidly as the Area increases and are almost inversely proportional to the increase in Area.

The results of similar experiments made by other investigators have been stated in many different ways, most of them however being the results of experiments involving some one particular application, so that the impirical formulæ as given by Prof. Lorber probably is more general and covers the subject more completely than do any or all of the others.

As the actual and relative degrees of accuracy which may be expected in any particular practical application of the Polar Planimeter has been stated in connection with the description of that particular application in the preceding chapters, it need not be repeated here, but enough has been said to prove the truth of the assertion made of the Planimeter at the beginning of our discussion that "a knowledge of its invaluable capabilities and of the enormous saving in time and labor effected by its use is the only requisite to make it the invaluable and inseparable co-laborer of the Engineer in almost every detail of his professional life.

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TABLES

Areas **DECIMALS OF AN INCH** **No. 1**

Scale				Vernier Unit		Test Plate		Otts		K. & E., No. 4230		Coradis Compensating			
No.	Ratio	Linear	Area	Actual	Relative	Area	Reading	Setting	Constant	Setting	Constant	Setting	Constant	Setting	Constant
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
	Ins.	Ins. Ft.	Sq. Ins. Sq. Ft.	Sq. Ins.	Sq. Ft.	Sq. Ft.	Units	½M/M'S	Units	½M/M S	Units	½M/M'S	Units	½M/M'S	Units
1	1:12	1 = 1	1=1	.01	.01	15.5	1550	217.6	19221	145.3	19308	212.6	25795	----	----
2	1:24	1 = 2	1=4	.0125	.05	62.0	1240	266.0	17868	92.6	18356	265.6	23683	----	----
3	1:36	1 = 3	1=9	.01	.01 Y'd	15.5 Yds	1550	217.6	19221	145.3	19308	212.6	25795	----	----
4	1:48	1 = 4	1=16	.0125	.2	248.0	1240	266.0	17868	92.6	18356	265.6	23683	----	----
5	1:60	1 = 5	1=25	.01	.25	387.5	1550	217.6	19221	145.3	19308	212.6	25795	----	----
6	1:72	1 = 6	1=36	.0111	.4	558.0	1395	239.0	18475	121.8	18760	236.1	24648	----	----
7	1:96	1 = 8	1=64	.0125	.8	992.0	1240	266.0	17868	92.6	18356	265.6	23683	----	----
8	1:120	1 = 10	1=100	.01	1.0	1550.0	1550	217.6	19221	145.3	19308	212.6	25795	----	----
9	1:144	1 = 12	1=144	.0111	1.6	2232.0	1395	239.0	18471	121.8	18760	236.1	24648	----	----
10	1:196	1 = 16	1=256	.01562	4.0	3968.0	992	327.3	17343	26.8	18194	332.0	22617	----	----
11	1:240	1 = 20	1=400	.0125	5.0	6200.0	1240	266.0	17868	92.6	18356	265.6	23683	----	----
12	1:300	1 = 25	1=625	.016	10.0	9687.0	968.7	334.8	17339	18.9	18218	340.1	22564	----	----
13	1:360	1 = 30	1=900	.0111	10.0	13950.0	1395	239.0	18475	121.8	18760	236.1	24648	----	----
14	1:480	1 = 40	1=1600	.0125	20.0	24800.0	1240	266.0	17868	92.6	18356	265.6	23683	----	----
15	1:600	1 = 50	1=2500	.01	25.0	38750.0	1550	217.6	19221	145.3	19308	212.6	25795	----	----
16	1:720	1 = 60	1=3600	.0111	40.0	55800.0	1395	239.0	18475	121.8	18760	236.1	24648	----	----
17	1:960	1 = 80	1=6400	.0125	80.0	99200.0	1240	266.0	17875	92.6	18356	265.6	23683	----	----
18	1:1000	1 = 83⅓	1=6946.38	.01432	100.0	107669.0	1076.7	302.8	17420	54.2	18162	304.4	22914	----	----
19	1:1200	1 = 100	1=10000	.01	100.0	155000.0	1550	217.6	19221	145.3	19308	212.6	25795	----	----

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Scale				Vernier Unit		Test Plate		Otts		K. & E., No. 4230		Coradis Compensating			
No.	Ratio	Linear	Area	Actual	Relative	Area	Reading	Setting	Constant	Setting	Constant	Setting	Constant	Setting	Constant
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
No.	Ft.	Ft. Ft.	Sq. Ft.	Sq. Ft.	Sq. Ft.	Sq. Ft.	Units	$\frac{1}{2}$M/M'S	Units	$\frac{1}{2}$M/M'S	Units	$\frac{1}{2}$M/M'S	Units	$\frac{1}{2}$M/M'S	Units
1	1:10	1 = 10	1 = 100	.0144	.01	10.764	1076.4	303.5	17458	52.6	18010	305.9	22881	----	----
2	1:20	1 = 20	1 = 400	.0144	.04	43.056	1076.4	303.5	17458	52.6	18010	305.9	22881	----	----
3	1:30	1 = 30	1 = 900	.0144	.09	96.876	1076.4	303.5	17458	52.6	18010	305.9	22881	----	----
4	1:40	1 = 40	1 = 1600	.0144	.16	172.224	1076.4	303.5	17458	52.6	18010	305.9	22881	----	----
5	1:50	1 = 50	1 = 2500	.0144	.25	269.10	1076.4	303.5	17458	52.6	18010	305.9	22881	----	----
6	1:60	1 = 60	1 = 3600	.0144	.36	387.50	1076.4	303.5	17458	52.6	18010	305.9	22881	----	----
7	1:70	1 = 70	1 = 4900	.0144	.49	527.43	1076.4	303.5	17458	52.6	18010	305.9	22881	----	----
8	1:80	1 = 80	1 = 6400	.0144	.64	688.69	1076.4	303.5	17458	52.6	18010	305.9	22881	----	----
9	1:90	1 = 90	1 = 8100	.0144	.81	871.88	1076.4	303.5	17458	52.6	18010	305.9	22881	----	----
10	1:100	1 = 100	1 = 10000	.0144	1.00	1076.41	1076.4	303.5	17458	52.6	18010	305.9	22881	----	----

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Scale				Vernier Unit		Test Plate		Ott's		K. & E., No. 4230		Coradis Compensating			
No.	Ratio	Linear	Area	Actual	Relative	Area	Reading	Setting	Constant	Setting	Constant	Setting	Constant	Setting	Constant
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
		Ins. Ft.	Sq. Ins. Sq. Ft.	Sq. Ins.	Sq. Ft.	Sq. Ft.	Units	½M/M'S	Units	½M/M'S	Units	½M/M'S	Units	½M/M'S	Units
1	1:192	1/16 = 1	1=256.0	.01562	4.0	3968.0	992	327.3	17343	26.8	18194	332.0	22617	----	----
2	1:96	1/8 = 1	1= 64.0	.01562	1.0	992.0	992.0	327.3	17343	26.8	18194	332.0	22617	----	----
3	1:64	3/16 = 1	1= 28.4408	.01406	.4	440.83	1102.2	297.1	17519	59.6	18174	298.8	22994	----	----
4	1:48	1/4 = 1	1= 16.0	.0125	.2	248.00	1240	266.0	17868	92.6	18356	265.6	23683	----	----
5	1:32	3/8 = 1	1= 7.1111	.01406	.1	110.22	1102.2	297.1	17519	59.6	18174	298.8	22994	----	----
6	1:24	1/2 = 1	1= 4.0	.0125	.05	62.00	1240	266.0	17868	92.6	18356	265.6	23683	----	----
7	1:19.2	5/8 = 1	1= 2.56	.01562	.04	39.68	992	327.3	17343	26.8	18194	332.0	22617	----	----
8	1:16	3/4 = 1	1= 1.7777	.01406	.025	27.555	1102.2	297.1	17519	59.6	18174	298.8	22994	----	----
9	1:12	1 = 1	1= 1.0	.01	.01	15.5	1550	217.6	19221	145.3	19308	212.6	25795	----	----
10	1:8	1½ = 1	1= 0.4444	.01575	.007	6.888	984	329.8	17341	24.2	18198	334.7	22602	----	----
11	1:6	2 = 1	1= 0.25	.01	.0025	3.875	1550	217.6	19221	145.3	19308	212.6	25795	----	----
12	1:4	3 = 1	1= 0.1111	.0108	.0012	1.722	1435	233.3	18674	128.4	18844	229.5	24941	----	----
13	1:2	6 = 1	1= 0.0278	.0108	.0003	0.43	1435	233.3	18674	128.4	18844	229.5	24941	----	----
No.	In. Ft.	Sq.In. Sq. Ft.	Sq. In. Sq.Yds.	Sq. Ins.	Cu. Yds.	Cu. Yds.	Units	½M/M'S	Units	½M/M'S	Units	½M/M'S	Units	½M/M'S	Units
1	1/16=1	1=256.0	1=9.481472	.01265	.12	146.9632	1224.7	268.8	17803	89.4	18321	268.9	23601	----	----
2	1/8=1	1= 64.0	1=2.370368	.01265	.03	36.7407	1224.7	268.8	17803	89.4	18321	268.9	23601	----	----
3	3/16=1	1= 28.4408	1=1.053362	.01519	.16	16.32711	1020.5	318.2	17337	35.8	18194	323.2	22705	----	----
4	1/4=1	1= 16.0	1=0.592592	.0135	.008	9.18514	1148.1	285.4	17584	71.5	18216	287.2	23220	----	----
5	3/8=1	1= 7.1111	1=0.263374	.01519	.004	4.08223	1020.5	318.2	17337	35.8	18194	323.2	22705	----	----
6	1/2=1	1= 4.0	1=0.148148	.0135	.002	2.29629	1148.1	285.4	17584	71.5	18216	287.2	23220	----	----
7	5/8=1	1= 2.56	1=0.094815	.01265	.012	1.46963	1224.7	286.8	17803	89.4	18321	268.9	23601	----	----
8	3/4=1	1= 1.7777	1=0.065841	.01519	.001	1.02052	1020.5	318.2	17337	35.8	18194	323.2	22705	----	----
9	1 =1	1= 1.0	1=0.037037	.0135	.0005	0.57407	1148.1	285.4	17584	71.5	18216	287.2	23220	----	----
10	1½=1	1= 0.4444	1=0.016459	.01458	.0024	0.25511	1063.0	307.0	17434	48.8	18160	309.9	22789	----	----
11	2 =1	1= 0.25	1=0.009259	.0108	.0001	0.14351	1435.1	233.3	18674	128.4	18844	229.5	24941	----	----
12	3 =1	1= 0.1111	1=0.004115	.01458	.00006	0.06378	1063.0	307.0	17434	48.8	18160	309.9	22789	----	----
13	6 =1	1= 0 0278	1=0.001029	.01458	.00015	0.01595	1063.0	307.0	17434	48.8	18160	309.9	22789	----	----

Note.—Length of section or thickness of wall is always taken as 1 foot.
Table can be at once used for vols. in cu. ft. as the No. of sq. ft. in any surface is evidently the same as the No. of cu. ft. when the section is taken as 1 ft. in length.

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Scale					Vernier Unit		Test Plate		Otts.		K. & E., No. 4230		Coradis Compensating			
No.	Ratio	Linear		Area	Actual	Relative	Area	Reading	Setting	Constant	Setting	Constant	Setting	Constant	Setting	Constant
1	2	3		4	5	6	7	8	9	10	11	12	13	14	15	16
No	Ins. Ins.	Ins.	Miles	Sq. In. Acres	Sq. In.	Acres	Acres	Units	½M/M'S	Units	½M/M'S	Units	½M/M'S	Units	½M/M'S	Units
1	1:528	120	= 1	1= .0444	.01575	.007	.6888	984	329.8	17341	24.2	18198	334.7	22602	----	----
2	1:792	80	= 1	1= .10	.01	.001	1.55	1550	217.6	19221	145.3	19308	212.6	25795	----	----
3	1:1050	60	= 1	1= .1777	.01406	.025	2.7555	1102.2	297.1	17519	59.6	18174	298.8	22994	----	----
4	1:1584	40	= 1	1= .40	.0125	.005	6.20	1240	266.0	17868	92.6	18356	265.6	23683	----	----
5	1:1980	32	= 1	1= .625	.016	.01	9.687	968.7	334.8	17339	18.9	18218	340.1	22564	----	----
6	1:3168	20	= 1	1= 1.6	.0125	.02	24.80	1240	266.0	17868	92.6	18356	265.6	23683	----	----
7	1:3960	16	= 1	1= 2.5	.01	.025	38.75	1550	217.6	19221	145.3	19308	213.6	25795	----	----
8	1:5280	12	= 1	1= 4.4444	.01575	.07	68.8888	984	329.8	17341	24.2	18198	334.7	22602	----	----
9	1:6336	10	= 1	1= 6.4	.0125	.08	99.2	1240	266.0	17868	92.6	18356	265.6	23683	----	----
10	1:7920	8	= 1	1=10.0	.01	.10	155.0	1550	217.6	19221	145.3	19308	212.6	25795	----	----
11	1:10560	6	= 1	1=17.777	.01406	.25	275.55	1102.2	297.1	17519	59.6	18174	298.8	22994	----	----
12	1:12672	5	= 1	1=25.0	.01562	.40	396.8	992	327.3	17343	26.8	18194	332.0	22617	----	----
13	1:15840	4	= 1	1=40.0	.01	.40	620.0	1550	217.6	19221	145.3	19308	212.6	25795	----	----
14	1:21120	3	= 1	1=71.1111	.01406	1.0	1102.22	1102.2	297.1	17519	59.6	18174	298.8	22994	----	----
15	1:25344	2½	= 1	1= .16 Sq. M.	.0125	.20	2.48 Sq. M.	1240	266.0	17868	92.6	18356	265.6	23683	----	----
16	1:12672	5	= 1	1= .04 "	.01	.0004	.62 "	1550	217.6	19221	145.3	19308	212 6	25795	----	----
17	1:6336	10	= 1	1= .01 "	.01	.001	.155 "	1550	217.6	19221	145.3	19308	212.6	25795	----	----

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Scale				Vernier Unit		Test Plate		Otts		K. & E., No. 4230		Coradis Compensating			
No.	Ratio	Linear	Area	Actual	Relative	Area	Reading	Setting	Constant	Setting	Constant	Setting	Constant	Setting	Constant
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
No.	Ft. Ft.	In. Ft.	Sq. In. Acres	Sq. Ins.	Acres	Acres	Units	½M/M'S	Units	½M/M'S	Units	½M/M'S	Units	½M/M'S	Units
1	1:120	1 = 10	1= .002295	.01307	.00003	.03558	1186.1	277.1	17699	80.5	18274	277.1	23386	....	....
2	1:240	1 = 20	1= .009182	.01307	.00012	.01423	1186.1	277.1	17699	80.5	18274	277.1	23386	....	....
3	1:360	1 = 30	1= .02066	.01452	.0003	.32024	1067.4	305.5	17390	50.0	18163	308.6	22860	....	....
4	1:480	1 = 40	1= .03673	.0136	.0005	.56931	1138.7	287.5	17569	69.2	18200	289.2	23155	....	....
5	1:600	1 = 50	1= .057	.01307	.0007	.8835	1262.2	261.9	17954	97 3	18400	260.9	23814	....	....
6	1:720	1 = 60	1= .08264	.01452	.0012	1.28092	1067.4	305.5	17390	50.0	18163	308.6	22860	....	....
7	1:960	1 = 80	1= .14692	.0136	.002	2.2774	1138.7	287.7	17569	69.2	18200	289.2	23155	....	....
8	1:1200	1 = 100	1= .22957	.01307	.003	3.5583	1186.1	277.1	17699	80.5	18274	277.1	23386	....	....
9	1:792	1 = 66	1= .1	.01	.001	1.55	1550	217.6	19221	145.3	19308	212.6	25795	....	....
10	1:1584	1 = 132	1= .4	.01	.004	6.20	1550	217.6	19221	145.3	19308	212.6	25795	....	....
11	1:2400	1 = 200	1= .91828	.01307	.012	14.23318	1186.1	277.1	17699	80.5	18274	277.1	23386	....	....
12	1:3600	1 = 300	1=2.066	.01452	.003	32.0045	1067.4	305.5	17390	50.0	18163	308.6	22860	....	....
13	1:4800	1 = 400	1=3.67312	.01307	.018	56.93272	1186.1	277.1	17699	80.5	18274	277 1	23386	....	....

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Scale				Vernier Unit		Test Plate		Otts		K. & E., No. 4230		Coradis Compensating			
No.	Ratio	Linear	Area	Actual	Relative	Area	Reading	Setting	Constant	Setting	Constant	Setting	Constant	Setting	Constant
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
No.	$^{M}/_{M}$ $^{M}/_{M}$	$^{M}/_{M}$ $^{M}/_{M}$	Sq $^{M}/_{M}$ Sq.$^{M}/_{M}$	Sq. $^{M}/_{M}$	Sq. $^{M}/_{M}$	Sq. $^{M}/_{M}$	Units	½$^{M}/_{M}$'S	Units	½$^{M}/_{M}$'S	Units	½$^{M}/_{M}$'S	Units	½$^{M}/_{M}$'S	Units
1	1:1	1 = 1	1 = 1	10.	10.	10000	1000	324.1	17305	29.4	18409	329.4	22641	----	----
2	1:2	1 = 2	1 = 4	10.	40.	40000	1000	324.1	17305	29.4	18409	329.4	26641	----	----
3	1:5	1 = 5	1 = 25	10.	250.	250000	1000	324.1	17305	29.4	18409	329.4	22641	----	----
4	1:10	1 = 10	1 = 100	10.	1000.	1000000	1000	324.1	17305	29.4	18409	329.4	22641	----	----
No.	M : M	$^{M}/_{M}$ $^{M}/_{M}$	Sq.$^{M}/_{M}$ Sq.$^{M}/_{M}$	Sq. $^{M}/_{M}$	Sq. M	Sq. M	Units	½$^{M}/_{M}$'S	Units	½$^{M}/_{M}$'S	Units	½$^{M}/_{M}$'S	Units	½$^{M}/_{M}$'S	Units
5	1:20	1 = 20	1 = 400	10.	.1	100	1000	324.1	17305	29.4	18409	329.4	22641	----	----
6	1:25	1 = 25	1 = 625	8.	.5	625	1250	264.0	17899	94.8	18317	263.5	23736	----	----
7	1:50	1 = 50	1 = 2500	10.	2.5	2500	1000	324.1	17305	29.4	18409	329.4	22641	----	----
No.	M : M	$^{M}/_{M}$ $^{M}/_{M}$	Sq. $^{M}/_{M}$ Sq. M	Sq. $^{M}/_{M}$	Sq. M	Sq. M	Units	½$^{M}/_{M}$'S	Units	½$^{M}/_{M}$'S	Units	½$^{M}/_{M}$'S	Units	½$^{M}/_{M}$'S	Units
8	1:100	1 = 100	1 = .1	10.	.1	100	1000	324.1	17305	29.4	18409	329.4	22641	----	----
9	1:250	1 = 250	1 = .0625	8.	.5	625	1250	264.0	17899	94.8	18317	263.5	23736	----	----
10	1:500	1 = 500	1 = .25	10.	2.5	2500	1000	324.1	17305	29.4	18409	329.4	22641	----	----
No.	M : M	Metres	Sq. Metres	Sq. $^{M}/_{M}$	Sq. M.	Sq. M	Units	½$^{M}/_{M}$'S	Units	½$^{M}/_{M}$'S	Units	½$^{M}/_{M}$'S	Units	½$^{M}/_{M}$'S	Units
11	1:1000	1= 1.0	1 = 1.0	10.0	10.0	10000	1000	324.1	17305	29.4	18409	329.4	22641	----	----
12	1:1250	1= 1.25	1 = 1.5625	6.4	10.0	15625	1562.5	216.3	19299	146.9	18317	210.8	25885	----	----
13	1:2500	1= 2.5	1 = 6.25	8.0	5.0	62500	1250	264.0	17899	94.8	18317	263.5	23736	----	----
14	1:4000	1= 4.0	1 = 16.0	6.25	100.0	160000	1600	211.8	19495	151.9	19832	205.9	26185	----	----
15	1:5000	1= 5.0	1 = 25.0	8.0	200.0	250000	1250	264.0	17899	94.8	18317	263.5	23736	----	----
16	1:10000	1=10.0	1 = 100.0	10.0	1000.0	1000000	1000	324.1	17305	29.4	18409	329.4	22641	----	----
17	1:20000	1=20.0	1 = 400.0	10.0	4000.0	4000000	1000	324.1	17305	29.4	18409	329.4	22641	----	----
18	1:360	1= .36	1 = .1296	7.712	1.0	1295	1296	255.6	18075	104.0	18480	254.2	24019	----	----
19	1:480	1= .48	1 = .2304	8.68	2.0	2304	1152	284.6	17602	72.5	18546	285.9	23215	----	----
20	1:1200	1= 1.20	1 = 1.44	6.944	10.0	14400	1440	232.4	18686	129.2	18920	228.7	24977	----	----

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VOLUMES OF SINGLE PRISMOIDS

Scale				Vernier Unit		Test Plate			Otts		K. & E., No. 4230		Coradis Compensating			
No.	Linear	Area	Volume	Actual	Relative	Area	Volume	Reading	Setting	Constant	Setting	Constant	Setting	Constant	Setting	Constant
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
No.	In. Ft.	Sq. In. / Sq. Ft.	Sq. In. Cu. Yds.	Sq. In.	Cu. Yds.	Sq. Ft.	Cu. Yds.	Units	½M/M'S	Units	½M/M'S	Units	½M/M'S	Units	½M/M'S	Units
1	1=4	1=16	1= 9.87648	.01265	0.125	248.0	153.0854	1224.7	268.8	17803	89.4	18321	268.9	23601	....	....
2	1=5	1=25	1= 15.4321	.01458	0.225	387.5	239.1975	1063.1	307.0	17434	48.8	18160	309.9	22789	....	....
3	1=6	1=36	1= 22.2222	.01395	0.31	558.0	344.4444	1111.1	294.3	17488	62.2	18170	296.5	23044	....	....
4	1=8	1=64	1= 39.5062	.01265	0.5	992.0	612.350	1224.7	268.8	17803	89.4	18321	268.9	23601	....	....
5	1=10	1=100	1= 61.72839	.01458	0.9	1550.0	956.790	1063.1	307.0	17434	48.8	18160	309.9	22789	....	....
6	1=12	1=144	1= 88.8888	.01406	1.25	2232.0	1377.770	1102.2	297.1	17519	59.6	18174	298.8	22994	....	....
7	1=16	1=256	1=158.0240	.01265	2.0	3968.0	2449.400	1224.7	268.8	17803	89.4	18321	268.9	23601	....	....
8	1=20	1=100	1=246.9120	.01458	3.6	6200.0	3827.36	1063.1	307.0	17434	48.8	18160	309.9	22789	....	....
9	1=24	1=576	1=355.5532	.01406	5.0	8928.0	5511.0715	1102.2	297.1	17519	59.6	18174	298.8	22994	....	....

Note.—Length of Prismoids are taken as 100′ = distance between cross sections.

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VOLUMES OF CONTINUOUS PRISMOIDS

Scale				Vernier Unit		Test Plate			Otts		K. & E., No. 4230		Coradis Compensating			
No.	Linear	Area	Volume	Actual	Relative	Area	Volume	Reading	Setting	Constant	Setting	Constant	Setting	Constant	Setting	Constant
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
No.	In. Ft.	Sq. In. Ft.	Cu. Yds.	Sq. In.	Cu. Yds.	Sq. Ft.	Cu. Yds.	Units	½M/M'S	Units	½M/M'S	Units	½M/M'S	Units	½M/M'S	Units
1	1=4	1=16	1= 39.5062	.01265	.5	248.0	612.236	1224.7	268.8	17803	89.3	18321	268.9	23601	----	----
2	1=5	1=25	1= 61.7285	.0162	1.0	387.5	956.790	956.79	337.6	17277	14.7	18242	344.0	22525	----	----
3	1=6	1=36	1= 88.8888	.01406	1.25	558.0	1377.750	1102.2	297.1	17519	59.7	18174	298.8	22994	----	----
4	1=8	1=64	1= 158.0249	.01265	2.0	992.0	2449.380	1224.7	268.8	17803	89.3	18321	268.9	23601	----	----
5	1=10	1=100	1= 246.9140	.0162	4.0	1550.0	3827.160	956.79	337.6	17277	14.7	18242	344.0	22525	----	----
6	1=12	1=144	1= 355.5561	.01406	5.0	2232.0	5511.00	1102.2	297.1	17519	59.7	18174	298.8	22994	----	----
7	1=16	1=256	1= 632.1198	.01265	8.0	3968.0	9798.00	1224.7	268.8	17803	89.3	18321	268.9	23601	----	----
8	1=20	1=400	1= 987.6560	.0162	16.0	6200.0	15308.42	956.79	337.6	17277	14.7	18242	344.0	22525	----	----
9	1=21	1=576	1=1422.2246	.01406	20.0	8928.0	22044.12	1102.2	297.1	17519	59.7	18174	298.8	22994	----	----

Note.—All cross sections are taken as 100′ apart.

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Scale				Vernier Unit		Test Plate			Otts		K. & E., No. 4230		Coradis Compensating			
No.	Linear	Area	Volume	Actual	Relative	Area	Volume	Reading	Setting	Constant	Setting	Constant	Setting	Constant	Setting	Constant
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
No.	In. Ft.	Sq. In. Sq. Ft.	Cu. Yds.	Sq. In.	Cu. Yds.	Sq. Ft.	Cu. Yds.	Units	½M/M'S	Units	½M/M'S	Units	½M/M'S	Units	½M/M'S	Units
1	1=4	1=16	1= .395062	.01265	.005	248.0	6.12236	1224.7	268.8	17803	89.3	18321	268.9	23601	----	----
2	1=5	1=25	1= .617285	.0162	.01	387.5	9.5679	956.79	337.6	17277	14.7	18242	344.0	22525	----	----
3	1=6	1=36	1= .888889	.01406	.0125	558.0	13.7775	1102.2	297.1	17519	59 7	18174	298.8	22994	----	----
4	1=8	1=64	1= 1.580249	.01265	.02	992.0	24.4938	1224.7	268.8	17803	89.3	18321	268.9	23601	----	----
5	1=10	1=100	1= 2.469140	.0162	.04	1550.0	38.2716	956.79	337.6	17277	14.7	18242	344.0	22525	----	----
6	1=12	1=144	1= 3.555561	.01406	.05	2232.0	55.1100	1102.2	297.1	17519	59.7	18174	298.8	22994	----	----
7	1=16	1=256	1= 6.321198	.01265	.08	3968.0	97.9800	1224.7	268.8	17803	89.3	18321	268.9	23601	----	----
8	1=20	1=400	1= 9.876560	.0162	.16	6200.0	153.0842	956.79	337.6	17277	14.7	18242	344.0	22525	----	----
9	1=24	1=765	1= 14.222246	.01406	.20	8928.0	220.4412	1102.2	297.1	17519	59.7	18174	298.8	22994	----	----
10	1=30	1=900	1= 22.22226	.0162	.36	13950.0	344.4444	956.79	337.6	17277	14.7	18242	344.0	22525	----	----
11	1=40	1=1600	1= 39.5062	.01265	.50	24800.0	612.236	1224.7	268.8	17803	89.3	18321	268.9	23601	----	----
12	1=50	1=2500	1= 61.7285	.0162	1.0	38750.0	956.790	956.79	337.6	17277	14.7	18242	344.0	22525	----	----
13	1=60	1=3600	1= 88.8888	.01406	1.25	55800.0	1377.75	1102.2	297.1	17519	59.7	18242	344.0	22525	----	----
14	1=80	1=6400	1=158.0249	.01265	2.0	99200.0	2449.38	1224.7	268.8	17803	89.3	18321	268.9	23601	----	----

Note.—Vertical distance between contours is always taken as 1 foot.

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Scale				Vernier Unit		Test Plate			Otts		K. & E., No. 4230		Coradis Compensating			
No.	Linear	Area	No. Brick	Actual	Relative	Area	No. Brick	Reading	Setting	Constant	Setting	Constant	Setting	Constant	Setting	Constant
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
No.	In. Ft.	Sq. In. Ft.	No. Brick	Sq. In.	No. Brick	Sq. Ft.	No. Brick	Units	½M/M'S	Units	½M/M'S	Units	½M/M'S	Units	½M/M'S	Units
1	1/16=1	1=256.0	1=2176.0	.01478	70.0	3968.0	73408.0	1048.7	310.1	17372	44.6	17823	314.0	22781	....	....
2	1/8=1	1= 64.0	1=1184.0	.01351	16.0	992.0	18352.0	1147	285.6	17601	71.3	18212	287.2	23200	....	....
3	3/16=1	1= 28.4408	1= 526.15	.0152	8.0	440.83	8155.59	1019.55	318.3	17281	35.7	19180	323.1	22705	....	....
4	1/4=1	1= 16.0	1= 296.0	.01351	4.0	248.0	4588.0	1147	285.6	17601	71.3	18212	287.2	23200	....	....
5	3/8=1	1= 7.1111	I= 131.55	.0152	2.0	110.25	2039.1	1019.55	318.3	17281	35.7	19180	323.1	22705	....	....
6	1/2 1	1= 4.0	1= 74.0	.01351	1.0	62.0	1147.0	1147	285.6	17601	71.3	18212	287.2	23200	....	....
7	5/8=1	1= 2.56	1= 47.36	.01478	.7	39.68	734.08	1048.7	310.1	17372	44.6	17823	314.0	22781	....	....
8	3/4=1	1= 1.7777	1= 32.874	.0152	.5	27.54	509.59	1019.5	318.3	17281	35.7	19180	323.1	22705	....	....
9	1 =1	1= 1.0	1= 18.5	.01351	.25	15.5	286.75	1147	285.6	17601	71.3	18212	287.2	23200	....	....
10	1½=1	1= .4444	1= 8.222	.0152	.125	6.888	127.44	1019.5	318.3	17281	35.7	19180	323.1	22705	....	....
11	2 =1	1= .25	1= 4.625	.01351	.0625	3 875	71.6875	1147	285.6	17601	71.3	18212	287.2	23200	....	....
12	3 =1	1= .1111	1= 2.055	.0146	.03	1.722	31.8602	1062	306.5	17383	48.5	18159	310.1	22829	....	....
13	6 =1	1= .0278	1= 0.5137	.01362	.007	.4304	7.9623	1137.5	286.4	17479	69.0	18193	289.6	23155	....	....

Note.—In all cases the section measured is taken as being 1 foot in length or thickness.

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WEIGHTS OF WROUGHT IRON AND OTHER METAL SECTIONS

Scale				Vernier Unit		Test Plate			Otts		K. & E., No. 4230		Coradis Compensating			
No.	Linear	Area	Volume	Actual	Relative	Area	Weight	Reading	Setting	Constant	Setting	Constant	Setting	Constant	Setting	Constant
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
No.	In. Ft.	Sq. In. Sq. Ft.	Sq. In. Cu. Ft.	Sq. In.	Lbs.	Sq. Ft.	Lbs.	Units	½M/M'S	Units	½M/M'S	Units	½M/M'S	Units	½M/M'S	Units
1	1/8=1	1=64.	1=5.3333	.01562	40.0	992.0	39680.0	992	327.3	17343	24.2	18194	332.0	22617	....	....
2	3/16=1	1= 2.3701	1=2.3701	.01406	16.0	440.83	17635.2	1102.2	297.1	17519	59.6	18174	298.8	22994	....	....
3	1/4=1	1= 1.3333	1=1.3333	.01562	10.0	248.0	9920.0	992	327.3	17343	24.2	18194	332.0	22617	....	....
4	3/8=1	1= 7.1111	1=0.5926	.01406	4.0	110.25	4408.9	1102.2	297.1	17519	59.6	18174	298.8	22994	....	....
5	1/2=1	1— 4.0	1=0.3333	.0125	2.0	62.0	2480.0	1240	266.0	17868	92.6	18356	265.6	23683	....	....
6	5/8=1	1= 2.56	1=0.2133	.015136	1.55	39.68	1587.2	1024	317.0	17331	37.1	18163	321.7	22707	....	....
7	3/4=1	1= 1.7777	1=0.1481	.01406	1.0	27.54	1102.2	1102.2	297.1	17519	59.6	18174	298.8	22994	....	....
8	1 =1	1= 1.0	1=0.0833	.0125	.5	15.50	620.0	1240	266.0	17868	92.6	18356	265.6	23683	....	....
9	1 1/2=1	1= 0.4444	1=0.0370	.01406	.25	6.888	275.55	1102.2	297.1	17519	59.6	18174	298.8	22994	....	....
10	2 =1	1= 0.25	1=0.0208	.01	.1	3.875	155.0	1550	217.6	19221	145.3	19308	212.6	25795	....	....
11	3 =1	1= 0.1111	1=0.0093	.0135	.06	1.722	68.888	1148	285.4	17584	71.5	18216	287.2	23220	....	....
12	4 =1	1= 0.0625	1=0.0052	.01	.05	.9687	38.75	1550	217.6	19221	145.3	19308	212.6	25795	....	....
13	6 =1	1= 0.0278	1=0.0023	.0135	.15	.4304	17.236	1148	285.4	17584	71.5	18216	287.2	23220	....	....

Note.—In all cases the section measured is taken as being 1 inch in length or thickness.

Gauge Points of Various Metals.

Wrt. Iron.. 1.00 Cast Iron.. 0.93 Cast Steel.. 1.02 Steel Plates.. 1.04 Copper.. 1.15 Brass.. 1.09 Lead.. 1.47 Zinc.. 0.92

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Scale				Vernier Unit		Test Plate			Otts		K. & E., No. 4230		Coradis Compensating			
No.	Linear	Area	Volume	Actual	Relative	Area	Volume	Reading	Setting	Constant	Setting	Constant	Setting	Constant	Setting	Constant
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
No.	In. Ft	Sq In. Sq. Ft.	Sq. In. Galls.	Sq. Ins.	Galls.	Sq. Ft.	Galls.	Units	½M/M'S	Units	½M/M'S	Units	½M/M'S	Units	½M/M'S	Units
1	1=1	1=1	1= 4.987	.014036	.07	15.5	77.2985	1104.26	295.7	17468	60.3	18176	298.3	23003	----	----
2	1=2	1=4	1= 19.948	.01504	. .	62.0	309.194	1030.6	315.1	17334	39.2	18161	319.6	22730	----	----
3	1=4	1=16	1= 79.792	.01253	1.0	248.0	1236.77	1236.77	266.59	17854	92.9	18480	266.2	23620	----	----
4	1=5	1=25	1= 124.675	.01203	1.5	387.5	1932.46	1288.31	256.9	18036	102.5	18466	255.8	24248	----	----
5	1=6	1=36	1= 179.532	.01392	2.5	558.0	2782.75	1113.1	293.5	17509	63.5	18130	295.8	23035	----	----
6	1=8	1=64	1= 319.168	.01253	4.0	992.0	4947.10	1236.77	266.6	18036	92.9	18480	266.2	23620	----	----
7	1=10	1=100	1= 498.700	.014036	7.0	1550.0	7729.85	1104.26	295.7	17468	60.3	18176	298.3	23003	----	----
8	=12	1=144	1= 718.128	.01392	10.0	2232.0	11130.98	1113.1	293.5	17509	63.5	18130	295.8	23035	----	----
9	1=16	1=256	1= 1276.672	.01253	16.0	3968.0	19788.42	1236.77	266.6	18036	92.9	18480	266 2	23620	----	----
10	1=20	1=400	1= 1994.800	.01504	30.0	6200.0	30919.40	1030.6	315.1	17334	39.2	18161	319.6	22730	----	----
11	1=24	1=576	1= 2872.512	.01392	40.0	9687.0	44523.94	1113.1	293.5	17509	63.5	18130	295.8	23035	----	----
12	1=30	1=900	1= 4488.300	.01114	50.0	13950.0	69568.65	1391.37	239.5	18449	121.2	18748	237.6	24689	----	----
13	1=40	1=1600	1= 7979.200	.01253	100.0	24800.0	123677.6	1236.77	266.6	17854	92.9	18480	266.2	23620	----	----
14	1=50	1=2500	1=12467.0	.01203	150.0	38750.0	193246.2	1288.31	256.9	18036	102.5	18466	255.8	24248	----	----
15	1=60	1=3600	1=17953.2	.01392	250.0	55800.0	278274.6	1113.1	293.5	17509	63.5	18130	295.8	23035	----	----
16	1=80	1=6400	1=31916.8	.01253	400.0	99200.0	494710.4	1236.77	266.6	18036	92.9	18480	266.2	23620	----	----
17	1=100	1=10000	1=49870.0	.014036	700.0	155000.0	772985.0	1104.26	295.7	17468	60.3	18176	298.3	23003	----	----

Note.—In all cases contours are taken as being 1 foot apart vertically.

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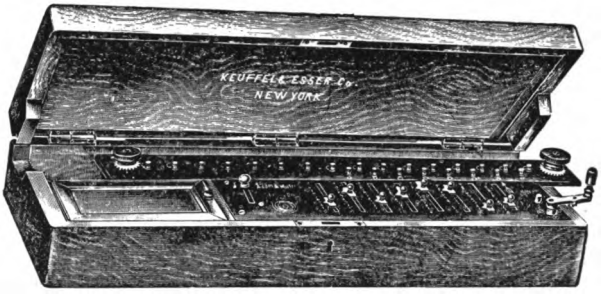
KEUFFEL & ESSER CO., - NEW YORK.

RECKONING MACHINES.



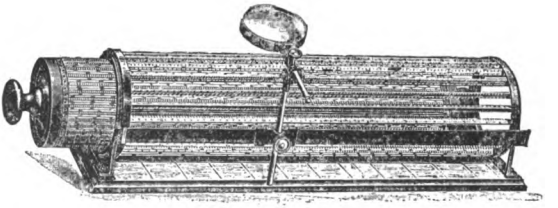
No. 4006.

Calculations, such as Addition, Subtraction, Multiplication, Division, Squaring, Cubing, Extracting of Square Roots, etc., etc., can be performed with rapidity and unfailing accuracy and without mental effort by means of this machine. The tedious work of reckoning is reduced to setting the figure discs or shifters and turning the crank handle, a simple mechanical operation. The machine will multiply two factors, each of which may have as many figures as there are grooves in the machine.

The machine is mechanically perfect and built in the most substantial manner so that it will retain its efficiency and accuracy for a very long time. There are a great many of these machines in use in public and private offices and scientific laboratories here and in Europe, and they are in all cases giving the greatest satisfaction.

		EACH
4005.	Reckoning Machine, 6 grooves, 12 holes in upper row	$193 25
4006.	" " 8 " 16 " " " "	241 50
4007.	" " 10 " 20 " " " "	338 25

THACHER'S CALCULATING INSTRUMENT.



No. 4013.

4012. Thacher's Calculating Instrument, for performing a great variety of useful calculations with unexampled rapidity and accuracy; cylinder 18 inches, in polished mahogany box, with directions..... Each, $35 00

4013. Thacher's Calculating Instrument, but with 3-inch reading glass sliding on brass bar, adjustable to any part of the instrument and for focus..... " 45 00

4013A. Directions for Thacher's Calculating Instrument. " 1 00

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SLIDE RULES.

Goodchild Mathematical Chart.

No. 4019, stout paper	each,	$ .75
" 4019 A, mounted on board	"	2.75
" 4019 B, triangular rule, for Marking and Adding	"	5.00

Fuller's Spiral Slide Rule.

No. 4015	"	30.00

Charpentier Calculator.

No. 4020	"	5.00

Boucher Calculator.

No. 4022	"	14.00
" 4022 A	"	12.50
" 4023	"	19.00
" 4024	"	8.50

Gunter Slide Rule.

No. 4028	"	2.00



K & E Mannheim Slide Rules, Patent Adjustable

No. 4030, 5 in., brass indicator	each,	$4.50
" 4031, " glass "	"	4.50
" 4032, " brass " with lens	"	6 00
" 4040, 10 in., " "	"	4.50
" 4041, " glass "	"	4.50
" 4050, 20 in., brass "	"	12.50
" 4051, " glass "	"	12.50

Mannheim Slide Rules, Favorite.

No. 4054, 10 in., glass indicator	"	3.00
" 4056, " brass "	"	2.75

Duplex Slide Rules.

No. 4060, 5 in., brass indicator	"	6.50
" 4061, " glass "	"	8.00
" 4065, " brass " 2 interchangeable slides	"	8.00
" 4066, " glass " 2 " "	"	9 50
" 4070, 10 in., brass "	"	6.50
" 4071, " glass "	"	8.00
" 4075, " brass " 2 interchangeable slides	"	8.00
" 4076, " glass " 2 " "	"	9 50
" 4078, 20 in., brass "	"	16.50
" 4079, " glass "	"	18.00
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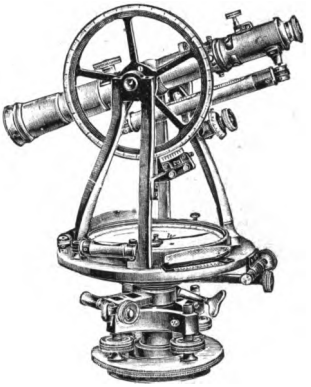

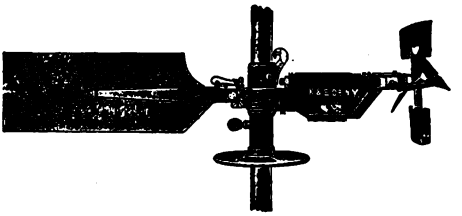

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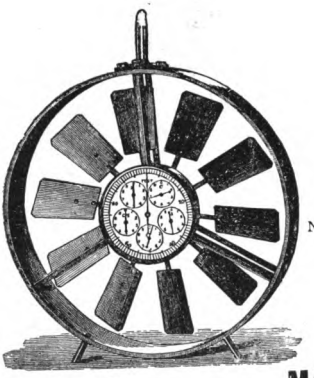

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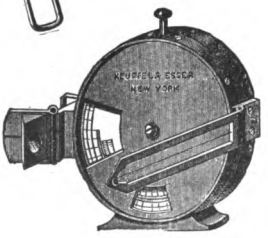

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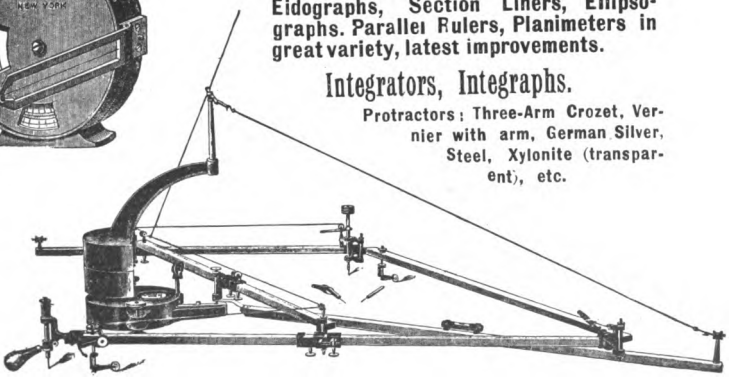

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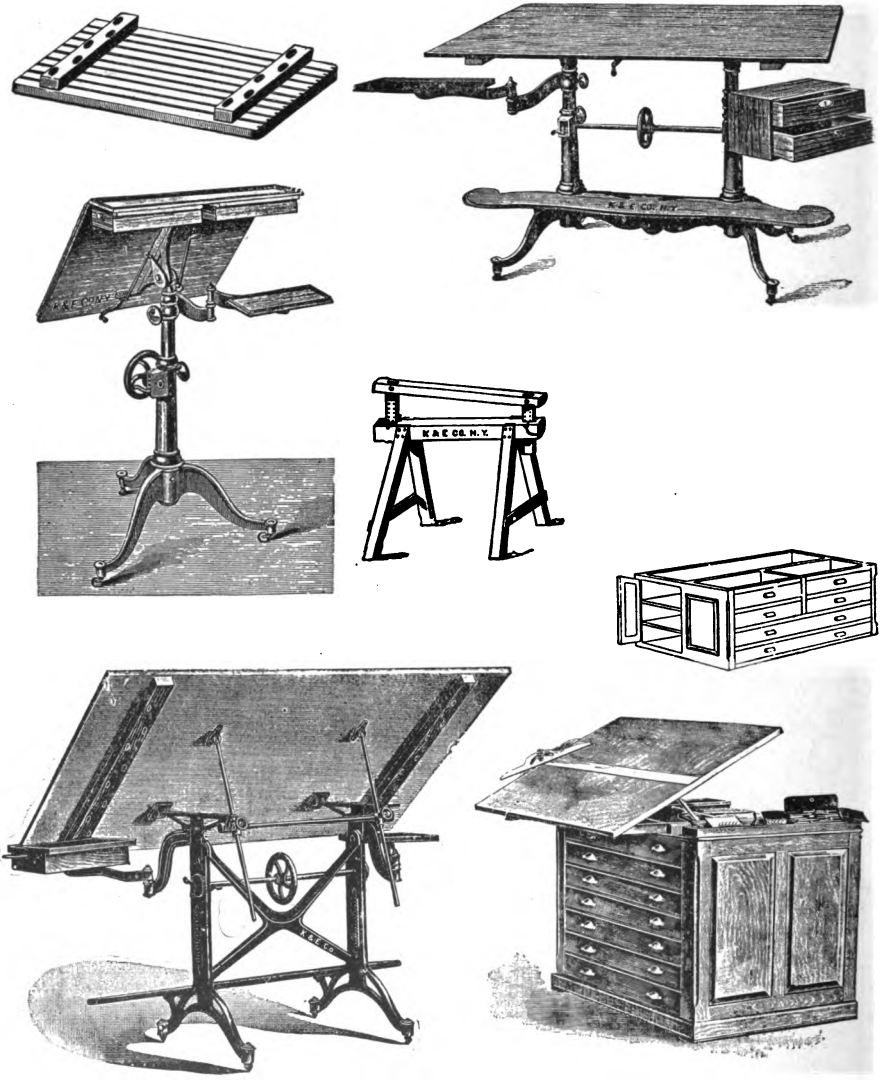

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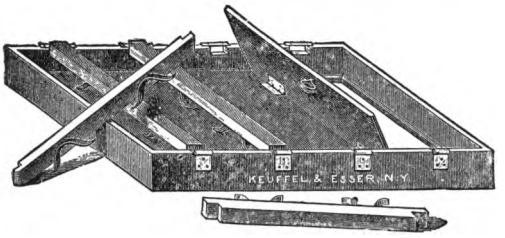

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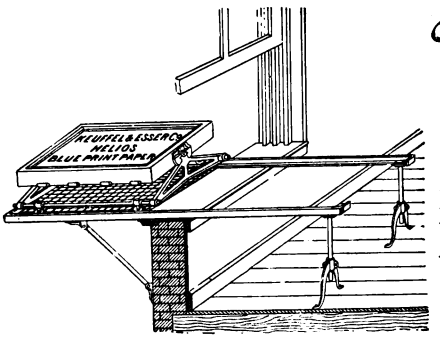

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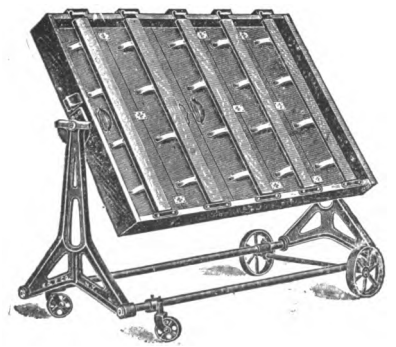

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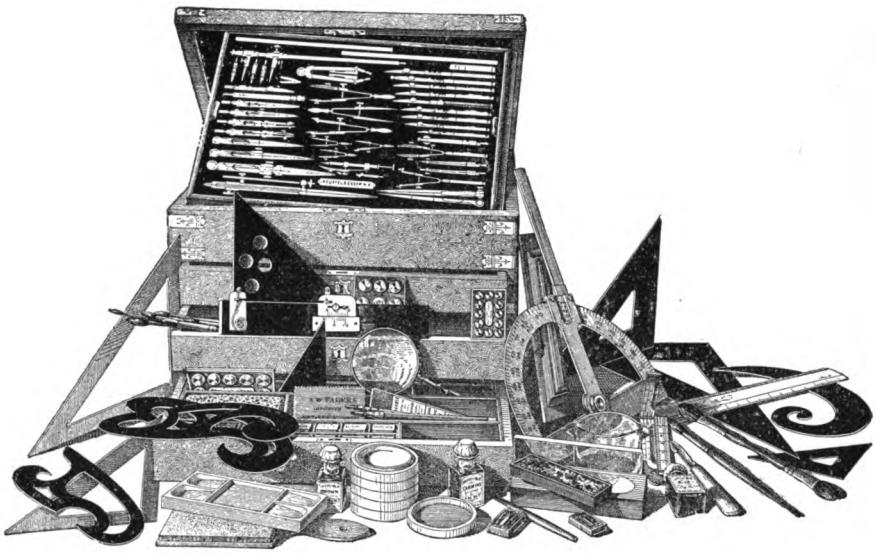

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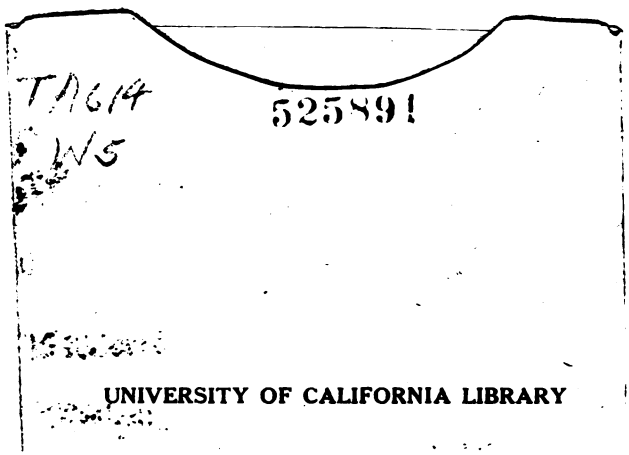
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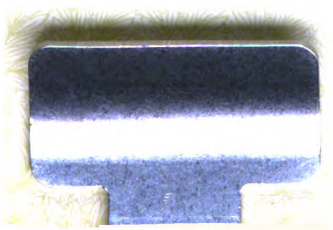

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